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A mathematical analysis of the bioenergetics of hurdling

A.J. WARD-SMITH

Department of Sport Sciences, Brunel University, Osterley Campus, Borough Road, Isleworth, Middlesex TW7 5DU, UK

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A mathematical model of the bioenergetics of running has been extended to apply to hurdling. This has been achieved by two principal modifications. First, a new term has been added to the energy equation to account for the potential energy required to negotiate each hurdle. Secondly, the term describing the degradation of mechanical energy to thermal energy has been modified to account for the adjustments in stride pattern necessary to negotiate the hurdles. A comparison of predicted and actual running times of elite athletes was made and good levels of agreement were achieved.

Keywords: Athletics, bioenergetics, biomechanics, hurdling.

Introduction

Hurdling can be regarded as a specialized form of sprinting in which athletes negotiate a hurdle by exaggerating their normal stride pattern. In the men's 110 m and women's 100 m hurdles, about one stride in every four is exaggerated, whereas in the men's and women's 400 m hurdles – depending on the natural stride length of the athlete – one stride in anything from 13 to 19 is exaggerated.

The starting point for the present paper is the mathematical analysis of the running performance of male athletes (Ward-Smith, 1985a). Recently, the method has been reviewed, been found to be robust and been applied to female runners by Ward-Smith and Mobey (1995). The theoretical approach is based on a consideration of the dominant energy transformations that take place during running, as shown in Fig. 1. Second-order effects, such as the small-scale cyclical variations of energy level associated with the stride pattern, are ignored on order-of-magnitude grounds.

Qualitative aspects of hurdling

To maintain a good horizontal speed in hurdling, the obstacles must not only be cleared, but be cleared in such a way that the clearance process has only a minimal effect on the horizontal running speed. The take-off distance in front of the hurdles depends on the height, leg length, speed, flexibility and technique of the athlete. Expert hurdlers instinctively modify the take-off distance in accordance with their horizontal velocity over the ground. Thus, as an athlete's speed increases over the first few hurdles, the take-off distance increases accordingly. Equally, adjustments have to be made over the final few hurdles whenever fatigue reduces the athlete's speed. As the athlete prepares to clear the hurdle, the lead leg and the torso move forwards and towards each other so that the athlete's centre of mass is raised above the ground by the smallest amount consistent with clearing the hurdle. This effect is enhanced by some athletes who, when clearing the hurdle, lower their head as far as possible. Ideally, to maintain the fastest horizontal speed, the athlete skims over the hurdle with the centre of mass as low as possible and consistent with a balanced landing on the track. As soon as the lead leg passes the obstacle, the hurdler begins to bring it back into contact with the ground, ensuring that the aerial distance after the hurdle is as short as possible. Speed is only gained or maintained through propulsive action when the hurdler is in contact with the ground; minimizing the hurdling distance and duration therefore leads to a maximization of the proportion of time devoted to sprinting. The

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action of the trailing leg during hurdling is rather different from that in normal sprinting. Whereas in sprinting it passes vertically beneath the body, in hurdling the trailing leg is abducted away from the running plane to avoid contact with the hurdle. As a consequence of this action, the centre of mass is raised by a smaller amount than would otherwise be the case. By minimizing the vertical movement of the centre of mass, the amount of potential energy that has to be generated at each hurdle is minimized.

It is known from steady-state treadmill tests that there is an essentially linear relationship between running speed and oxygen consumption (Margaria, 1976; Davies, 1980). It follows (Ward-Smith, 1984) that the rate of increase of energy dissipated as heat in running is directly proportional to velocity under steady-state conditions. Ward-Smith (1984, 1985a) has shown, by extrapolation, that this relationship can also be applied to the acceleration phase at the start of sprinting. In hurdling, the normal sprinting action is compromised, adjustments being made by the athlete to both the stride length and vertical displacement of the centre of mass to accommodate the height and spacing of the hurdles. Because the stride pattern is compromised, it follows that the rate of degradation of mechanical energy into thermal energy, or, expressed another way, the rate of heat production, is greater in hurdling than in sprinting.

Summarizing the above discussion, from an energy standpoint there are two principal differences between hurdling and normal sprinting. In hurdling, potential energy has to be supplied to the centre of mass at each hurdle over and above the amount associated with normal running. Also, the stride pattern is distorted relative to that in running. Ultimately, both of these factors contribute to an increase in the energy dissipated as heat.

Mathematical analysis

The preceding discussion has shown that hurdling can be regarded as a specialized form of sprinting adapted to the clearance of obstacles. Here we begin the analysis of hurdling by using the mathematical model of running, based on energy considerations, established by Ward-Smith (1985a). Modifications appropriate to hurdling will then be introduced. The analysis considers energy changes associated with the motion of the centre of mass of the runner; for distances greater than a few stride lengths, the cyclical variations associated with the stride pattern are ignored on order-of-magnitude grounds. The energy exchanges are summarized in Fig. 1.

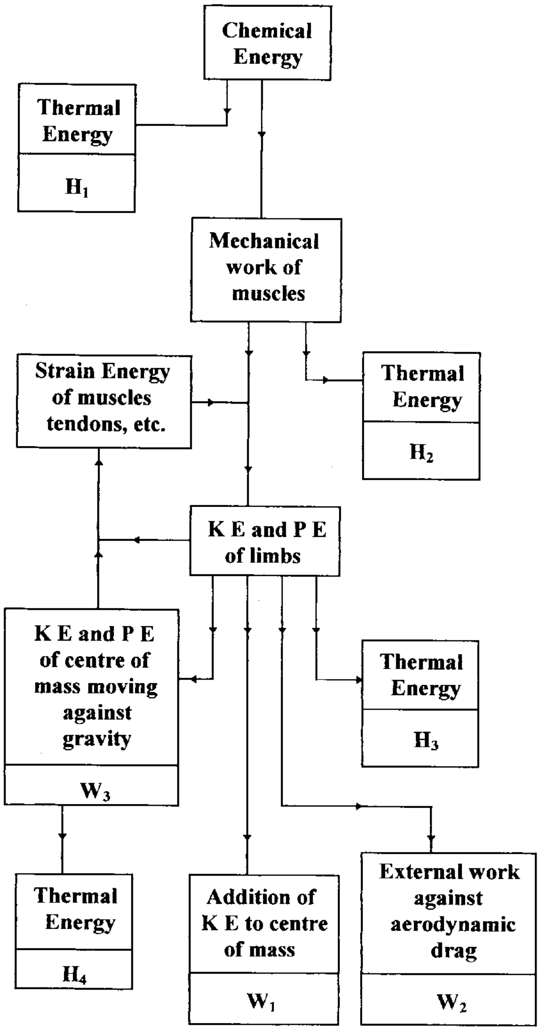

Figure 1 Representation of energy exchanges occurring during running.

Energy balance during running

The chemical energy released, C, is equal to the sum of the external work done on the centre of mass of the hurdler, W, plus the mechanical energy converted into thermal energy, H. The overall energy balance is given by:

$$C = H + W \tag{1}$$

Differentiation of the energy balance with respect to time, t, results in the corresponding power equation, given by:

$$\frac{dC}{dt} = \frac{dH}{dt} + \frac{dW}{dt} \tag{2}$$

The individual contributions to the power equation are as follows.

The rate of chemical energy conversion is, following Ward-Smith (1985a):

$$\frac{\mathrm{d}C}{\mathrm{d}t} = P_{\max}\exp(-\lambda t) + R[1 - \exp(-\lambda t)] \qquad (3)$$

where $P_{\max}$ represents the maximum power available from the anaerobic mechanism, R is the maximum aerobic power and λ is a parameter governing the rate of release of chemical energy.

The sum of the four contributions H_1, H_2, H_3 and H_4 shown in Fig. 1 is denoted by H. The overall rate of degradation of mechanical energy into heat is given by:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = Av \qquad (4)$$

where v is velocity and A is a parameter governing the rate of dissipation of mechanical energy.

The work done by adding kinetic energy to the centre of mass will be denoted by W_1 and the work done against aerodynamic drag by W_2. Then, the rate at which the horizontal component of kinetic energy is added to the runner of mass m is given by:

$$\frac{\mathrm{d}W_1}{\mathrm{d}t} = mv\,\frac{\mathrm{d}v}{\mathrm{d}t} \qquad (5)$$

The rate of working against aerodynamic drag, for still air conditions, is:

$$\frac{\mathrm{d}W_2}{\mathrm{d}t} = Dv = \frac{1}{2}\rho v^3 SC_{\mathrm{D}} = \frac{1}{2}\rho v^3 A_{\mathrm{D}} \qquad (6)$$

where D is the aerodynamic drag experienced by the body, ρ is the air density, S is the projected frontal area of the body, C_{D} is the drag coefficient and A_{D} ($=SC_{\mathrm{D}}$) is the drag area.

Substitution of the individual components into equation (2) yields the full form of the power equation for running, which is:

$$P_{\max}\exp(-\lambda t) + R[1 - \exp(-\lambda t)] = Av + \frac{1}{2}\rho v^3 SC_{\mathrm{D}} + mv\,\frac{\mathrm{d}v}{\mathrm{d}t} \qquad (7)$$

In this formulation of the power equation, the relatively small effects of the vertical movement of the runner's centre of mass in the earth's gravitational field are absorbed into the dissipation term, Av, as explained in Fig. 1.

The energy equation is derived by integration of equation (7) with respect to t between the limits $t=0$, $x=0$ and $t=T$, $x=X$, and is

$$\frac{P_{\max}}{\lambda}[1 - \exp(-\lambda T)] + R[T - \frac{1}{\lambda}(1 - \exp(-\lambda T))] = AX + K\int_0^T v^3\,\mathrm{d}t + \frac{1}{2}mV^2 \qquad (8)$$

where

$$K = \frac{\rho SC_{\mathrm{D}}}{2} \qquad (9)$$

Modifications for hurdling

For hurdling, the analysis developed for running needs to be modified in two ways. First, it is useful to consider explicitly an additional component in the power and energy equations which accounts for the changed vertical movement of the body's centre of mass associated with the negotiation of the hurdles. Just as in running, over the race as a whole, the net vertical displacement of the centre of mass is almost zero; this term is therefore ultimately manifested as a contribution to the degradation term. However, it is convenient to represent the term explicitly in the analysis of hurdling. Secondly, it must be recognized that, to negotiate the hurdles, athletes are forced to depart from their optimal stride pattern adopted in running, and as a consequence there is an increased degradation of mechanical energy into heat associated with this effect.

By conservation of energy, the vertical component of kinetic energy when taking off to clear the hurdle is converted to potential energy because of the gain in height of the centre of mass of the athlete. Thus:

$$\frac{1}{2}mV_{\mathrm{V}}^2 = mgh \qquad (10)$$

where h is the vertical displacement of the centre of mass relative to its nominal horizontal level, and V_{V} is the vertical velocity component at take-off. If the number of hurdles is denoted by N_{H}, then the additional energy transferred to potential energy in clearing the hurdles is $N_{\mathrm{H}}mgh$. Whereas the terms in equation (7) – the power equation for running – are continuous, the contributions to the power and energy equations from the hurdling action are discontinuous. To establish the power equation for hurdling, we average the hurdling terms over the entire race. A power term associated with the potential energy used to clear the hurdles may be defined by $\mathrm{d}W_3/\mathrm{d}t$ and this is given by:

$$\frac{\mathrm{d}W_3}{\mathrm{d}t} = N_{\mathrm{H}}\frac{mgh}{T} \qquad (11)$$

The dissipation term for hurdling will be different from that for running and can be derived in the following way. It is inevitable that, to a greater or lesser extent, a hurdler has to modify the stride pattern not only in those strides used to clear the hurdles but in others in the approach and landing as well. As a basis for developing the theoretical work, we assume that a hurdler progresses through a race: (1) by advancing using a normal running action, and (2) by modifying the normal running action in a certain number of strides that is proportional to the number of hurdles cleared. Let A and A_H represent the rate of degradation of mechanical energy into thermal energy for normal running strides and hurdling strides, respectively. We also define N_T and N, respectively, as the total number of strides and the number of normal running strides in a hurdle race. Hence:

$$N_T = N + N_H \tag{12}$$

We introduce the symbol α to represent the proportion of the total number of strides used for negotiating the hurdles: Thus:

$$\alpha = \frac{N_H}{N_T} \tag{13}$$

Then:

$$\frac{N}{N_T} = \frac{N_T - N_H}{N_T} = (1 - \alpha) \tag{14}$$

An average effective rate of energy degradation for the entire race A_{eff} can then be defined to satisfy the relation:

$$A_{eff} = (1 - \alpha)A + \alpha A_H \tag{15}$$

We can relate A and A_H by the expression:

$$A_H = \beta A \quad \text{where } \beta = f(\alpha, h) \tag{16}$$

The parameter β will have a magnitude greater than 1 and in principle depends, to a greater or lesser extent, upon the vertical displacement of the centre of mass of the athlete and upon the stride pattern required to negotiate the hurdles.

Hence for hurdling the energy equation becomes:

$$\frac{P_{max} - R}{\lambda}[1 - \exp(-\lambda T)] + RT = A_{eff}X + K\int_0^T v^3 dt + \frac{1}{2}mv^2 + N_H mgh \tag{17}$$

The corresponding power equation is:

$$P_{max}\exp(-\lambda t) + R[1 - \exp(-\lambda t)] = A_{eff}v + Kv^3 + mv\frac{dv}{dt} + \frac{N_H mgh}{T} \tag{18}$$

Equation (18) can be rewritten as:

$$\frac{dv}{dt} = \frac{1}{v}[(P^*_{max} - R^*)\exp(-\lambda t) + R^* - A^*_{eff}v - K^*v^3 - \frac{N_H gh}{T}] \tag{19}$$

where values shown with an asterisk (*) are values normalized with respect to body mass. Thus $R^* = R/m$, etc. Furthermore, x, v and t are related by:

$$\frac{dx}{dt} = v \tag{20}$$

The total energy contributions from the anaerobic and aerobic mechanisms, C_{an} and C_{aer} respectively, can be obtained by integration of the corresponding power terms with respect to time. Thus the total chemical energy converted at time t from rest is given by:

$$C = C_{an} + C_{aer} \tag{21}$$

where

$$C_{an} = \frac{P_{max}}{\lambda}[1 - \exp(-\lambda t)] \tag{22}$$

and

$$C_{aer} = R[t - \frac{1}{\lambda}(1 - \exp(-\lambda t))] \tag{23}$$

Application of analysis to hurdling

Data relevant to the solution of the derived equations are assembled in this section. Attention will be focused on the performance of elite athletes.

Position of the hurdles on the track

Over the past century, there have been a number of different race distances. Currently, the recognized outdoor hurdle events are the men's 110 m and 400 m and the women's 100 m and 400 m. The men's and women's indoor 50 m and 60 m hurdles events have recently been recognized by the International Amateur Athletic Federation (IAAF). Although the 3000 m steeplechase requires hurdles to be cleared, this event has additional features and, because it is radically different from the other events considered here, it has been excluded from the analysis. The number of hurdles in an event, and the positions of the hurdles on the track, are given in Table 1.

Typical stride patterns of elite athletes

The quantitative data on stride patterns reported here were obtained by observing videos of elite hurdlers in action; additional unpublished data were provided by Paul Grimshaw.

In the men's 110 m hurdles, most elite hurdlers adopt a stride pattern of eight strides to the first hurdle, although some exceptional athletes use seven; three strides are used between hurdles, with the fourth used to negotiate the hurdle, and the race is completed with six strides to the finish. The race therefore consists of about 51 strides, 10 of which are exaggerated by hurdle clearance, yielding a representative value for α of 10/51. In the men's 400 m hurdles, most hurdlers use about 22 strides to the first hurdle, 13 or 15 strides between hurdles – although some athletes are able to alternate their leading leg and thereby use 14 strides – and about 25 strides from the last hurdle to the finish. As fatigue sets in, the natural stride shortens and so the stride pattern in the early part of the race may be different from that adopted later in the race. For a total of 192 strides, α is evaluated as 10/192. In the men's 60 m hurdles, athletes usually adopt the same stride pattern as in the 110 m hurdles, with eight strides to the first hurdle and, subsequently, three strides between each hurdle. For this event, α is about 5/28. In the 50 m hurdles, the stride pattern is the same as in the 60 m event, with the exception of the number of strides used in the final distance. An approximate estimate for α is 4/24.

In the women's 100 m hurdles, the stride pattern is the same as that in the men's 110 m hurdles, with eight strides to the first hurdle and three strides in between, the fourth stride being used to negotiate the hurdle, and six strides to the finish, yielding a representative value for α of 10/51. In the women's 400 m event, the stride pattern usually consists of about 28 strides to the first hurdle, either 17 or 19 strides between the hurdles, depending on the physique and natural stride of the athlete, followed by about 26 strides to the finishing line. As in the men's event, the number of strides may increase towards the end of the race due to fatigue. A representative value for α is 10/221. In the women's 60 m hurdles, about eight strides are used to the first hurdle and three strides in between. A typical value for α is 5/28. Over 50 m, the stride pattern adopted is the same as for the 60 m hurdles, with the exception of the number of strides at the finish. A representative value for α is 4/25.

Biophysical data

Representative biophysical data describing an elite male athlete capable of running the 100 m in 10.23 s have been established by Ward-Smith (1985a). Marar and Grimshaw (1993) have reported that the main factor which determines performance at hurdling is sprinting speed. Therefore, the representative biophysical data previously derived for a male sprinter are directly applicable to the case of a male hurdler. Corresponding values for an elite female runner capable of sprinting the 100 m in 10.93 s have been evaluated by Ward-Smith and Mobey (1995). Again, it is appropriate to use biophysical data for elite sprinters to evaluate hurdling performance.

Tanner (1964) showed that the average height of male hurdlers in the 1960 Olympics was 183 cm. A

Table 1 Positions of hurdles

Event	Number of hurdles	Distance from start to first hurdle (m)	Distance between hurdles (m)	Distance from last hurdle to finish (m)
Men's 50 m	4	13.72	9.14	8.86
Men's 60 m	5	13.72	9.14	9.72
Men's 110 m	10	13.72	9.14	14.02
Men's 400 m	10	45.00	35.00	40.00
Women's 50 m	4	13.00	8.50	11.50
Women's 60 m	5	13.00	8.50	13.00
Women's 100 m	10	13.00	8.50	10.50
Women's 400 m	10	45.00	35.00	40.00

Table 2 Magnitudes of parameters for male and female hurdlers

Parameter	Male hurdler	Female hurdler
λ	$0.03\ \mathrm{s}^{-1}$	$0.03\ \mathrm{s}^{-1}$
P^*_{max}	$50.5\ \mathrm{W\ kg}^{-1}$	$47.2\ \mathrm{W\ kg}^{-1}$
R^*	$23.5\ \mathrm{W\ kg}^{-1}$	$21.8\ \mathrm{W\ kg}^{-1}$
A^*	$3.9\ \mathrm{J\ kg}^{-1}\ \mathrm{m}^{-1}$	$3.98\ \mathrm{J\ kg}^{-1}\ \mathrm{m}^{-1}$
K^*	$3.3 \times 10^{-3}\ \mathrm{m}^{-1}$	$3.6 \times 10^{-3}\ \mathrm{m}^{-1}$
H	1.83 m	1.73 m
A_{D}	$0.385\ \mathrm{m}^2$	$0.358\ \mathrm{m}^2$
m	70 kg	60 kg

representative value of 173 cm for the height of world-class female hurdlers has been obtained from unpublished data provided by Paul Grimshaw. A summary of the representative biophysical and other data for substitution in the computer calculations is given in Table 2.

Centre of mass

Measurements of the vertical displacement of the centre of mass of several athletes have been made from Grimshaw's data. Information was obtained by measurement from three-dimensional digital images created from videos taken at various athletics meetings. The whole body centre of mass positions were obtained by the usual segmentation method. Data on the clearance height of a number of athletes were available for the men's 110 m and the women's 110 m hurdle events only, and these are presented in Table 3. The representative clearance height used in the present calculations is the average of the data in Table 3, and these data were used for all events except the 400 m hurdles.

Table 3 Performance data for athletes in competition (from Marar and Grimshaw, 1993)

Event	Time (s)	Clearance (cm)
Men's 60 m	7.77	27
Men's 60 m	8.40	28
Men's 60 m	8.20	24
Men's 110 m	13.64	29
Men's 110 m	13.86	28
Men's 110 m	13.33	25
Men's 110 m	13.23	25
Women's 100 m	13.24	40
Women's 100 m	13.33	36
Women's 100 m	13.72	34
Women's 100 m	13.08	35
Women's 100 m	13.15	37
Women's 100 m	13.41	37

As no measurements had been made for either the men's or women's 400 m events, an estimate of the clearance was made; the estimated value of the clearance was in close accord with data reported by Kaufmann and Piotrowski (1976) for hurdlers of intermediate standard. The vertical displacement of the centre of mass of the hurdler was then calculated by adding the clearance height to the height of the hurdle and subtracting the height of the centre of mass above the ground during sprinting. Page (1978) quoted the results of earlier work he had undertaken which showed that the centre of mass of an adult male in a normal upright stance lies about 2.5 cm below his navel, or approximately 57% of his full height from the ground. A female's weight is distributed differently; she has a wider pelvis and, usually, narrower and lighter shoulders. Her centre of mass is nearer the ground and is about 55% of her full height from the ground (Page, 1978).

The height of the hurdles varies according to the event. It is therefore necessary to consider each event separately in evaluating the mathematical model. The height of the hurdles, the average clearance height and the vertical displacement of the centre of mass for each event are summarized in Table 4.

Effective rate of energy degradation

At this stage, the one unknown quantity required to solve equations (19) and (20) is a measure of the degradation of mechanical energy to thermal energy associated with hurdling. In principle, the modified stride pattern, the number of hurdles and the vertical movement of the centre of mass are factors that affect A^*_{eff}. Here it will be assumed that the effect of the vertical movement is much smaller than the effect associated with the changed stride pattern, and can be ignored on order-of-magnitude grounds. This assumption can be tested when predicted and actual results are

Table 4 Estimated vertical displacement of centre of mass

Event	Height of hurdle (m)	Clearance (m)	Displacement of centre of mass (m)
Men's 50 m	1.067	0.268	0.291
Men's 60 m	1.067	0.268	0.291
Men's 110 m	1.067	0.268	0.291
Men's 400 m	0.914	0.318	0.188
Women's 50 m	0.84	0.365	0.254
Women's 60 m	0.84	0.365	0.254
Women's 100 m	0.84	0.365	0.254
Women's 400 m	0.762	0.365	0.176

compared. It is evident that A^*_{eff} must exceed A^* and the assumption will be made that the excess is directly proportional to α. Thus A^*_{eff} is relatred to A^* by the expression:

$$A^*_{\text{eff}} = (1 + \alpha)A^* \tag{24}$$

This relationship corresponds to a value for β of 2; expressed another way, for the strides directly affected by the hurdling process, the efficiency of locomotion is half that for normal running.

Results and discussion

Equations (19) and (20) were solved to obtain $v = v(t)$ and $x = x(t)$ using a numerical scheme based on the fourth-order Runge-Kutta method. A time-step of 0.01 s was adopted, and programs previously written for male sprinting (Ward-Smith, 1985a) and female sprinting (Ward-Smith and Mobey, 1995) were used as a basis for the program development.

The program made use of representative values of the biophysical data contained in Table 2; data for h and α derived above and summarized in Table 5 were also used. Zero wind and a nominal sea level density for air of 1.21 kg m^{-3} were assumed. For all events, current world record times were used as a basis of comparison with the mathematical model.

Table 5 Input data used in the program to calculate hurdling performance

Event	h (m)	N_H	α
Men's 50 m	0.29	4	0.17
Men's 60 m	0.29	5	0.18
Men's 110 m	0.29	10	0.20
Men's 400 m	0.19	10	0.052
Women's 50 m	0.25	4	0.16
Women's 60 m	0.25	5	0.18
Women's 100 m	0.25	10	0.20
Women's 400 m	0.18	10	0.045

Comparisons of predicted and actual running times are shown in Table 6, for both outdoor and indoor events. It is perhaps helpful to mention that several factors can contribute to the differences between the predicted and actual times. First, there is the basic variability in performance between one individual athlete and another, with the consequence that world records do not fit some precise correlating equation. Secondly, the physiological parameters of individual athletes may depart to some extent from the representative data incorporated into the present calculations. Thirdly, how well the mathematical model works as a whole depends on how well the individual contributions to the energy balance are modelled. For example, there is evidence that a tail wind enhanced the world records in the men's 110 m event (wind +0.5 m s^{-1}) and the women's 100 m event (wind +0.7 m s^{-1}). The data in Table 6 are not adjusted for this effect and we shall return to this matter later. Overall, the differences in Table 6 between predicted and actual times were generally less than 2.5%, with the exception of the women's 100 m event, where the predicted time was some 6% greater than the world record. Bearing in mind the three broad sources of possible discrepancies discussed above, the general level of correlation for the other seven events is considered to be very good. We note at this stage that, in deriving an expression for A^*_{eff}, the effect of the vertical movement of the centre of mass was ignored compared to the effect of the modified stride pattern on order-of-magnitude grounds. This assumption is seen to be justified, as there is no substantial disparity between the actual and predicted times which can be systematically correlated with this factor.

A number of supplementary calculations has been made. The energy contributions from the anaerobic

Table 6 Comparison of predicted hurdling times with world record times

Event	Actual time, t_A (s)	Predicted time, t_P (s)	Δt $(t_A - t_P)$	$(\Delta t/t_A)$ %
Men's 50 m	6.25	6.10	+0.15	+2.4
Men's 60 m	7.30	7.24	+0.06	+0.8
Men's 110 m	12.91	13.19	−0.28	−2.2
Men's 400 m	46.78	47.01	−0.23	−0.5
Women's 50 m	6.58	6.44	+0.14	+2.2
Women's 60 m	7.69	7.70	−0.01	−0.2
Women's 100 m	12.21	12.94	−0.73	−6.0
Women's 400 m	52.61	51.63	+0.98	+1.9

Table 7 Contributions from the aerobic and anaerobic mechanisms

Event	Actual time (s)	Anaerobic (%)	Aerobic (%)
Men's 50 m	6.25	95.7	4.3
Men's 60 m	7.30	95.0	5.0
Men's 110 m	12.91	91.2	8.8
Men's 400 m	46.78	71.4	28.6
Women's 50 m	6.58	95.5	4.5
Women's 60 m	7.69	94.8	5.2
Women's 100 m	12.21	91.8	8.2
Women's 400 m	52.61	68.6	31.4

and aerobic mechanisms were evaluated by substituting the world record times in equations (21) and (23). The results are given in Table 7. These show that the power exerted throughout a hurdling event is predominantly from the anaerobic source, a result which is consistent with previous results for running (Ward-Smith, 1985a). These calculations are also consistent with data reported by Åstrand and Rodahl (1986), who have tabulated the approximate contributions from the aerobic and anaerobic processes under general conditions of maximal effort in exercise. Åstrand and Rodahl give the anaerobic-to-aerobic ratio as 85% at 10 s and 65–70% after 60 s, diminishing to 1% after 2 h.

Indoor hurdling events are held under conditions where air movements are small; in the 400 m outdoor events, the effects of wind tend to be self-cancelling and are quite small. However, performance in the men's outdoor 110 m hurdles and the women's outdoor 100 m hurdles is significantly influenced by wind conditions. To take account of wind effects, equation (6) was modified. The rate of working against aerodynamic drag is then given by:

$$\frac{dW_2}{dt} = Dv = \frac{1}{2}\,\rho v(v - V_W)^2 S C_D = \frac{1}{2}\,\rho v(v - V_W)^2 A_D \qquad (25)$$

where V_W represents the wind velocity, positive for a following wind.

Calculations of the effects of both head and following winds were made, and the results are given in Table 8. These calculations indicate that a following wind of 2 m s^{-1} confers an advantage of just over 0.2 s for both the men's 110 m and the women's 100 m events. These figures are very similar to those predicted for sprinting; for example, an advantage of 0.18 s has been computed for the men's 100 m sprint (Ward-Smith, 1985b).

Table 8 Predicted influence of head and tail winds on hurdling performance (a following wind is positive; a head wind is negative)

Event	Wind speed (m s^{-1})	Predicted time (s)
Men's 110 m	−2	13.46
	−1	13.32
	0	13.19
	1	13.07
	2	12.97
Women's 100 m	−2	13.22
	−1	13.07
	0	12.94
	1	12.83
	2	12.73

Table 9 Predicted influence of vertical movement of centre of mass h on hurdling time

Event	h (m)	Predicted time (s)
Men's 50 m	0.24	6.07
	0.29	6.10
	0.34	6.13
Men's 60 m	0.24	7.20
	0.29	7.24
	0.34	7.28
Men's 110 m	0.24	13.09
	0.29	13.19
	0.34	13.29
Men's 400 m	0.14	46.87
	0.19	47.01
	0.24	47.15
Women's 50 m	0.20	6.40
	0.25	6.44
	0.30	6.47
Women's 60 m	0.20	7.66
	0.25	7.70
	0.30	7.75
Women's 100 m	0.20	12.83
	0.25	12.94
	0.30	13.06
Women's 400 m	0.13	51.48
	0.18	51.63
	0.23	51.79

Returning again to the conditions under which the world records were set and using the method on which Table 8 is based, the mathematical model for the men's 110 m hurdles, with $V_W = 0.5$ m s^{-1}, predicts a time of 13.13 s; for the women's 100 m hurdles, with $V_W = 0.7$ m s^{-1}, a time of 12.86 s is predicted. The corresponding percentage differences between the predicted and actual world record times (see Table 6) are thereby reduced to −1.7% and −5.2%, respectively. These calculations do suggest that the world record of 12.21 s for the women's 100 m hurdles, established by Yordanka Donkova on 20 August 1988, is quite exceptional when compared against all the other world record hurdling performances, even after the effects of wind assistance are taken into account.

The results of the computation depend to some extent upon the representative values of the vertical displacement of the centre of mass, *h*. The values used are shown in Table 4, and are based on the assumed position of the centre of mass of the runner as well as the average results for clearance height set out in Table 3. Although in Table 3 there is no direct correspondence between clearance height and running times, the times cover a range of performances, some of which fall significantly short of world standard. As it is useful to have some insight into how variations in *h* affect the predicted time, calculations at 1 cm intervals in *h* were made; a selection of the results is set out in Table 9. A detailed inspection of the calculations showed that, over the range investigated, the predicted time varied linearly with *h*; small but significant benefits arise from controlling this effect during hurdling.

The effects on hurdling performance of a number of variables, including body mass (*m*), projected frontal area of the athlete (*S*), air density (ρ) and drag coefficient (C_D), are taken into account by the parameter K^*. Rather than investigate the separate effects of these are variables, particularly bearing in mind that *m* and *S* are interrelated, calculations were made to investigate the overall effect of K^* on hurdling time. The results are shown in Table 10. The effect of changing K^* on predicted time is small; typically, a 20% change in K^* produced a change in predicted time of 1% or less.

Table 10 Predicted influence of K^* on hurdling performance times (s)

	K^* (m^{-1})		
Event (m)	0.0030	0.0033	0.0036
Men's 50 m	6.08	6.10	6.11
Men's 60 m	7.22	7.24	7.26
Men's 110 m	13.14	13.19	13.24
Men's 400 m	46.75	47.01	47.27
	K^* (m^{-1})		
	0.0033	0.0036	0.0039
Women's 50 m	6.42	6.44	6.45
Women's 60 m	7.68	7.70	7.72
Women's 100 m	12.90	12.94	12.98
Women's 400 m	51.39	51.63	51.88

Conclusion

It has been shown that a mathematical model of running can be successfully adapted to the analysis of hurdling. This has been achieved by two principal modifications. First, a new term has been added to the power (or energy) equation to account for the vertical displacement of the centre of mass required to negotiate the hurdles. Secondly, the term expressing the rate of degradation of mechanical energy to thermal energy has been increased in magnitude to account for the effects of the adjustments in stride pattern to negotiate the hurdles. Good correlations of actual and predicted times for outdoor and indoor hurdles events were achieved.

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