INFLUENCE OF POLE LENGTH AND STIFFNESS ON THE ENERGY CONVERSION IN POLE-VAULTING

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Abstract—An impact process similar to pole-vaulting is studied, viz., the impact in a vertical plane between the bottom end of a slightly curved elastic bar (pole), with a point mass (vaulter) at the top end, and a rigid support (pole box). Before impact, the velocity of the pole and the vaulter forms a certain angle (take-off) with the horizontal ground. Finite element calculations of the trajectories of the vaulter are carried out, and a performance figure, defined as the ratio between the maximum potential energy of the vaulter and the initial kinetic energy of the vaulter and the pole, is determined as a function of dimensionless parameters. As the vaulter remains passive during the vault, in contrast to a real vaulter, this performance figure is also the efficiency of conversion of the initial kinetic energy to the achieved potential energy in the vault. It is shown that, under normal pole-vault conditions, there exists a maximum performance figure with respect to pole length and stiffness. For an initial velocity and a body mass which are representative of an elite pole-vaulter, the maximum performance figure 0.87 is obtained for a pole with length 5.5 m.

Keywords: Mechanics; Pole; Vault; Energy conversion.

NOMENCLATURE

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Subscripts

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INTRODUCTION

Pole-vaulting is a sport in which conversion of the initial kinetic energy of a running vaulter and his pole into potential energy of the vaulter, is of primary importance. This potential energy can be directly translated into vertical height, which is the desired result of the vault. The conversion of energy is achieved in two principal phases. In the first phase, the pole is bent and a large fraction of the kinetic energy is converted into elastic strain energy. In the second phase, the pole straightens out and a large fraction of the stored elastic strain energy is converted into potential energy of the vaulter. In addition to the initial kinetic energy, the vaulter supplies muscle work during the vault which contributes to the final potential energy. Clearly, the length and stiffness of the pole are key parameters for the energy conversion and therefore also for a successful vault.

Studies of the mechanics of pole vaulting have been made by several researchers, both experimentally and theoretically. Dillman and Nelson (1968) determined experimentally kinetic and potential energies of a pole vaulter during a vault. Hubbard (1980) studied a three-segment vaulter with internal muscle forces and determined vaulter trajectories for different initial velocities of the vaulter. Walker and Kirmse (1982) used a one-segment pendulum model of the vaulter to study the effect of pole stiffness. McGinnis (1984) made finite element simulations with different pole stiffnesses for a vaulter with a fixed scheme of motions. Bruff and Dapena (1985) used a ten-segment vaulter model and an elastica pole to simulate vaulting with different pole stiffnesses. McGinnis and Bergman (1986) made an experimental study of the internal forces and moments of a vaulter during a pole vault. The effects of take-off velocity and take-off angle were considered by Linthorne (1994) for a point-mass vaulter using a massless perfectly rigid pole.
In a recent paper (Ekevad and Lundberg, 1995) we studied a six-segment (seven-element) vaulter who is 'smart' in the sense that he uses his muscles, to the best of his ability, to move his body segments as closely as possible to a prescribed motion, e.g. that which is typical of a particular elite vaulter. In this paper, in contrast, we have chosen to study a vaulter who is passive during the vault and uses the simplest possible style. We consider a simple impact process with similarity to pole vaulting, viz., the impact in a vertical plane between the bottom end of a slightly curved elastic beam (pole), with a point mass (vaulter) at the top end, and a rigid support (pole box). Before impact, the velocity of the pole and the vaulter forms a certain angle (take-off) with the horizontal ground. Finite element calculations of the trajectories of the vaulter are carried out, and we determine a performance figure as the ratio between the maximum potential energy of the vaulter and the initial kinetic energy of the vaulter and the pole, as a function of dimensionless parameters.

We focus our interest on the influence of the length and stiffness of the pole and show that under normal pole vault conditions there exists a maximum performance figure, not much below unity, with respect to pole length and stiffness. In this way we get some idea about rules for optimum pole design. Also, by comparing with results from our previous study of a more sophisticated active vaulter, we get some idea about the importance of smart behaviour and muscle work during the vault.

THEORY

Model

The pole is a prebent bar with length $L$, radius of curvature $R$, prebend angle $\beta = L/R$, cross-sectional area $A$, area moment of inertia $I$, radius of gyration $R_I = (1/4)^{1/2}$.$^{2}$ Young's modulus $E$, Poisson's ratio $\nu$, and mass per unit length $m$. Thus, the pole has constant properties along its length. The contact between the bottom end of the pole and the pole box is represented by a fixed pin joint, and the top end of the pole is fixed to the vaulter, who is represented by a point mass $M$. The vaulter grip height is $H$, and the common initial velocity $V$ of the vaulter and the pole forms the take-off angle $\alpha$ with the horizontal ground. The acceleration due to gravity is $g$, and the motion of the pole and the vaulter takes place in a vertical plane; see Fig. 1.

The performance figure is defined as the ratio between the maximum potential energy of the vaulter and the initial kinetic energy of the vaulter and the pole; i.e.

$$\eta = \frac{M}{M + mL} \frac{g(Y_M - H)}{v^2},$$

where $Y_M$ is the maximum vertical height of the vaulter during the vault (regardless of the horizontal position where this maximum height is reached). As the vaulter here remains passive during the vault, the performance figure is also the efficiency of conversion of the initial kinetic energy to the achieved potential energy in the vault. For an elastic pole, the losses of energy which make the performance figure less than unity are the increase of potential energy of the pole in the gravitational field, the strain energy of the pole, and the kinetic energy of the vaulter and the pole when the vaulter is at maximum height.

An approximate upper limit for the vault height, due to the fact that the vaulter is attached to the pole, is $Y_M = L$. This gives the upper limit

$$\eta U = \frac{M}{M + mL} \frac{g(L - H)}{v^2/2},$$

of the performance figure.

For an active (Ekevad and Lundberg, 1995) or real vaulter, who performs muscle work during the vault, the performance figure is commonly larger than unity and cannot be interpreted as an efficiency.

Rigid pole

The case of a straight rigid pole is characterized by the pole length $L$, the pole mass per unit length $m$, the vaulter mass $M$, the vaulter grip height $H$, the take-off angle $\alpha$, the common initial velocity of the vaulter and the pole $V$, and the acceleration due to gravity $g$.

Just before impact between the bottom end of the pole and the pole box, the pole and the vaulter have the common translational velocity $V$ which forms the angle $\alpha$ with the horizontal ground as shown in Fig. 1. At the moment of impact, there is an immediate change of the translational motion of the pole and the vaulter into a rotational motion around the fixed pole bottom pin joint. The angular velocity $\omega$ is such that the angular momentum of the pole and the vaulter with respect to the pole bottom pin joint immediately after impact is the same as that immediately before impact, i.e.

$$\omega L^2(M + mL/3) = VL \sin(\alpha + \phi_0)(M + mL/2),$$

Fig. 1. Pole vault represented by point mass on a bar impacting a stiff support.
where $\phi_0 = \arcsin(H/L)$ is the initial angle of the pole to the ground. The immediate change in the motion of the pole and the vaulter at impact implies a corresponding immediate loss of kinetic energy. For an elastic pole and a deformable pole box, this loss corresponds to waves generated in the pole and to work performed on the pole box.

The maximum vault height, as long as this height does not exceed the length of the pole, can be determined from the condition that the rotational kinetic energy of the pole and the vaulter immediately after impact should be completely transformed into potential energy of the pole and the vaulter, i.e.

$$(M + mL/2)(Y_M - H) = (\omega^2 L^2/2)(M + mL/3). \quad (4)$$

The corresponding provisional performance figure, which does not take into account the limited length of the pole, can be obtained by eliminating $\omega$ from equations (3) and (4), solving for $g(Y_M - H)$, and substituting into equation (1). The result is

$$\eta_k = \frac{M}{M + mL/2} \frac{M + mL}{M + mL/3} \frac{\sin^2(x + \phi_0)}{3} \sin^2(\phi + \phi_0). \quad (5)$$

This quantity is less than unity and decreases with pole length $L$.

Taking into account the limitation of pole length expressed by equation (2), finally, we obtain the performance figure

$$\eta = \min(\eta_k, \eta_t) \quad (6)$$

for a vault with a rigid pole.

The optimal pole length, i.e., the one which gives the maximum performance figure, is determined from the condition

$$\eta_k = \eta_t, \quad (7)$$

which means that the vaulter reaches $Y_M = L$ at $\phi = \phi_M = 90^\circ$, and with zero velocity.

**Finite element model of elastic pole**

Simulations with an elastic pole were carried out with the finite element programme ABAQUS Version 4-7-22 (1988). This programme uses nonlinear, large-displacement dynamic beam theory to calculate the time history of the pole and the vaulter. The 20 two-node B21 beam elements are large-displacement beams with non-deforming cross-sections and they allow for transverse shear. The elements have both mass and rotary inertia, but the vaulter has only mass. The programme uses an implicit time-stepping method to solve the equations of motion. The convergence tolerance on force is set to 0.001 times the buckling force of the straight pole. The time steps are chosen by the programme and are varied in order to give rapid convergence. The number of time steps varies from about 100 to about 500 for each simulation. Typically, about one to five load balance iterations are performed in each time step to achieve convergence within the force tolerance specified above.

**Dimensionless parameters**

Dimensionless parameters are taken as

$$\theta = H/L, \quad \beta = L/R, \quad r_1 = R/L, \quad \nu = \frac{V}{(gL)^{1/2}}, \quad \mu = M/mL, \quad \kappa = E/\rho g L^3, \quad (8)$$

where $\theta$ is the dimensionless vaulter grip height, $\beta$ is the prependicular angle, $r_1$ is the dimensionless radius of gyration of the pole, $\nu$ is the dimensionless common initial velocity of the vaulter and the pole, $\mu$ is the dimensionless mass of the vaulter, and $\kappa$ is the dimensionless stiffness of the pole (related to its mass). For the special case of a straight rigid pole, the only dimensionless parameters are $h, x, \beta, \nu$ and $\mu$. The dimensionless time is taken to be $\tau = (gL)^{1/2} t$.

**SIMULATIONS AND RESULTS**

For the dimensionless parameters of equation (8) which involve the length $L$ of the pole, except $\beta$, we define dimensionless parameters $h_0, r_0, \nu_0, \mu_0$ and $\kappa_0$ corresponding to a fixed length $L_0$ of the pole. A dimensionless pole length is then defined as

$$\dot{\lambda} = L/L_0, \quad (10)$$

which gives

$$\frac{\dot{h}}{h_0} = \frac{\dot{\lambda}}{\lambda_0}, \quad \frac{\dot{r}_1}{r_1_0}, \quad \nu = \frac{\nu_0}{(\lambda)^{1/2}}, \quad \frac{\mu}{\mu_0} = \frac{\kappa}{\kappa_0} \frac{\lambda}{\dot{\lambda}}, \quad \dot{\lambda} = \frac{\lambda}{\dot{\lambda}_0}. \quad (11)$$

The prependicular angle $\beta$ is considered to be constant, which means that the prependicular radius $R$ is proportional to the length $L$ of the pole.

We now study the performance figure $\eta$ as a function of the dimensionless pole length $\dot{\lambda}$ and the dimensionless pole stiffness $\kappa_0$ for constant values of the other parameters $h_0, \beta, r_0, \nu_0, x_0 = v_0/\nu_0$ and $\mu_0$. The performance figure (1) is expressed in terms of dimensionless quantities as

$$\eta = \frac{\mu_0}{\mu_0 + \dot{\lambda}} \frac{\nu_0}{\nu_0 + \dot{\lambda}}, \quad (12)$$

where $Y_{\text{om}} = Y_M/L_0$. Similarly, the upper limit (2) of the performance figure is expressed as

$$\eta_u = \frac{\mu_0}{\mu_0 + \dot{\lambda}} \frac{\dot{h}_0}{\nu_0 + \dot{\lambda}}. \quad (13)$$

We choose a vaulter with mass $M = 75$ kg, grip height $H = 2.25$ m, take-off velocity $V = 8.5$ m s$^{-1}$, and take-off angle $x = 20^\circ$. Also, we choose a reference pole with length $L_0 = 5.0$ m, prependicular angle $\beta = 0.40$ rad, pole mass per unit length $m = 0.60$ kg m$^{-1}$, radius of gyration $R_1 = 15$ mm and Poisson’s ratio $\nu = 0.25$. These are realistic values for a professional pole-vaulter with an appropriate pole. The acceleration due to gravity is taken to be...
$g = 10 \text{ m s}^{-2}$. The resulting dimensionless parameters are: $h_0 = 0.45$, $b = 0.40$, $r_{o0} = 0.0050$, $\nu = 0.25$, $\phi = 20^\circ$, $r_0 = 1.2$, and $\mu_0 = 25$.

**Rigid pole**

The performance figure (5) can be expressed as

$$
\eta_R = -\frac{\mu_0 - \mu_0}{\mu_0 + \lambda/2} \sin^2(\pi + \phi_0),
$$

where

$$
\sin(\pi + \phi_0) = [h_0 \cos(\pi) + (\lambda^2 - h_0^2)^{1/2} \sin(\pi)]/\lambda.
$$

(15)

The performance figure $\eta$ for a rigid pole, determined from equations (6), (13–15), is shown in Fig. 2 as a function of the dimensionless length $\lambda$ of the pole. The optimal value of $\lambda$ is 0.88, and the maximum performance figure is 0.59.

**Elastic pole**

Trajectories of the pole and the vaulter were calculated for different combinations of dimensionless pole lengths and stiffnesses. Dimensionless pole lengths $\lambda$ from 0.80 to 1.25 in intervals of 0.05 were used, and about ten dimensionless pole stiffnesses $\kappa_0$ were considered for each dimensionless pole length. The maximum height of the vaulter for each vault was determined and used to calculate the performance figure according to equation (12).

![Fig. 2. Performance figure $\eta$ versus dimensionless pole length $\lambda$ for straight rigid pole.](image)

![Fig. 3. Performance figure $\eta$ versus dimensionless pole stiffness $\kappa_0$ and dimensionless pole length $\lambda$ for elastic pole. The curve on the surface shows where the angle-to-ground at the top position $\phi_0$ is 90°. The dashed curve represents the condition $H_0 = \kappa_0^0$ for Euler buckling.](image)
Figure 3 shows the performance figure $\eta$ as a function of the dimensionless pole length $\lambda$ and the dimensionless pole stiffness $k_0$. Figure 4 shows the vaulter trajectories for different pole stiffnesses for a short pole with dimensionless pole length $\lambda = 0.80$, while Fig. 5 shows vaulter trajectories for different pole stiffnesses for a long pole with $\lambda = 1.21$. Figure 6 shows the vaulter trajectory for the optimal combination of pole length $\lambda = 1.1$ and pole stiffness $k_0 = 3.7$.

**DISCUSSION**

A comparison between the weight of the vaulter and the Euler buckling load for a simply supported straight pole indicates what stiffness the pole must have in order to be able to straighten completely under the weight of the vaulter. Equality between the vaulter weight and the Euler buckling load gives the minimum pole stiffness

$$k_{OE} = \frac{\mu_0 \lambda^2}{\pi^2}. \quad (16)$$

The relation $k_0 = k_{OE}$ is represented by the dashed curve in Fig. 3. For $k_0 < k_{OE}$, which corresponds to the bottom corner in the figure, the pole is not able to straighten completely.

The performance figure shown in Fig. 3 has a global maximum of 0.87 for $\lambda = 1.1$ and $k_0 = 3.7$. It can be seen that there is an increasingly narrow ridge (indicated with a curve) on the surface leading down from the global maximum through increasing $\lambda$ and decreasing $k_0$. This ridge corresponds to conditions which give $\phi_m = 90^\circ$, so that the top position of the vaulter is reached vertically above the pole–ground joint. The narrow part of the ridge, which is in the domain $k_0 < k_{OE}$ where the pole is not able to straighten completely, corresponds to the slow partial straightening of long poles almost vertically from the pole–ground joint as illustrated in Fig. 5(b). The performance figure for very stiff poles ($k_0 \to \infty$) approaches that shown in Fig. 2 for a rigid pole.

Relatively short and stiff poles straighten almost completely, and the vault height may be limited by the pole

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Fig. 4. Deformed configuration of pole and position of vaulter at different times with time step between two consecutive configurations $\Delta T = 0.10$. (a) Short pole with low stiffness, $\lambda = 0.80$ and $k_0 = 3.0$; (b) short pole with high stiffness, $\lambda = 0.80$ and $k_0 = 5.0$.

Fig. 5. Deformed configuration of pole and position of vaulter at different times. (a) Long pole with low stiffness, $\lambda = 1.21$ and $k_0 = 3.0$. Time step between two consecutive configurations $\Delta T = 0.10$. (b) Long pole with optimal stiffness, $\lambda = 1.21$ and $k_0 = 3.4$. Time step between two consecutive configurations $\Delta T = 0.20$. (c) Long pole with high stiffness, $\lambda = 1.21$ and $k_0 = 5.0$. Time step between two consecutive configurations $\Delta T = 0.10$. 

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length. Thus, for \( \lambda < 0.9 \) and \( 3 < \kappa_0 < 5 \) the performance figure takes on values near the limit \( \eta_2 \) given by equation (13). As \( \lambda < \mu_0 \), this means that in Fig. 3 the performance figure is approximately represented by a sloping plane, parallel with the \( \kappa_0 \)-axis. For such poles, the stiffness \( \kappa_0 \) determines the angle to ground \( \phi = \phi_M \) at the top position (cf. Fig. 1) as can be seen in Fig. 4 for \( \lambda = 0.80 \) and two different stiffnesses. Higher stiffness results in smaller angle, and vice versa.

Trajectories for the case of a long pole, \( \lambda = 1.21 \), are shown in Fig. 5. A pole with low stiffness, see Fig. 5(a), straightens slowly and incompletely, and the vaulter does not reach a very high level. A pole with appropriate stiffness, [see Fig. 5(b)] reduces the vaulter’s horizontal speed to near zero just above the pole–ground joint and then straightens in the vertical direction. However, the force from the pole is not large enough to lift the vaulter to the full length of the pole, and the vaulter falls off horizontally. The top position in this case corresponds to the ridge in Fig. 3. A very stiff pole straightens too quickly, and the vaulter is returned horizontally and reaches his highest vertical position at an angle \( \phi_M < 90^\circ \) [see Fig. 5(c)]. Again, the vaulter does not reach a very high level.

Figure 6 shows a pole-vault with a performance figure near the global maximum. The vaulter reaches the top position at \( \phi_M = 90^\circ \) with a low horizontal velocity, and the pole is almost completely straightened. A shorter pole would not allow the vaulter to reach the optimum height, and a longer pole would not give enough force to lift the vaulter to the optimum height.

We have shown for a realistic case that a maximum in performance figure with regard to dimensionless pole length \( \lambda \) and dimensionless pole stiffness \( \kappa_0 \) exists. The optimal dimensionless pole length is \( \lambda = 1.1 \) which corresponds to a pole length \( L \) of 5.5 m with our choice of parameters. This means a slightly longer pole than commonly used by elite pole vaulters today (about 5.2 m). The passive vaulter used in this work achieves a maximum performance figure \( \eta \) of 0.87, while for an active vaulter we have obtained a maximum performance figure of 1.27 (Ekevad and Lundberg, 1995). This comparison illustrates the significance of muscle work in pole vaulting and underlines that interpretations of results obtained with passive vaulters must be made with care.

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REFERENCES


