Dependence of release variables in the shot put

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Abstract

When the shot is released above a horizontal plane, range from this point depends on release height, speed and angle. Measured distance is the sum of this range and horizontal distance of the release point from the throwing circle edge. Optimal release conditions can be calculated only if the dependence of release velocity on other variables, due to thrower limitations, is known. Experiments on two shot-putters investigated the hypothesis that there are constraint relationships among these four release parameters. A variable scaling scheme, using measurement of impact point and the known magnitude of $g$, corrected 2D data from one camera for out-of-plane motion and yielded accurate estimates of release parameters. Multivariate regression analyses determined approximate constraint surfaces limiting performance. Achievable release speed decreases with increasing release angle at about 1.7 $\text{m/s}/\text{rad}$ and decreases with increasing release height at about 0.8 $\text{m/s}/\text{m}$, with only small differences in sensitivities between the throwers. Horizontal release distance also decreases with increasing release angle at about 1.7 m/rad and increases with increasing release height at about 1.3 m/m, again with only small differences between the two throwers. Optimal release conditions producing maximum range for a particular athlete can be determined using similar constraints for that athlete. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The shot put has been studied extensively, both analytically and experimentally. Theoretical studies and subsequent reviews (Garfoot, 1968; Lichtenberg and Wills, 1978; Zatsiorsky et al., 1981; Trowbridge and Paish, 1981; Burghes et al., 1982; Townsend, 1984; Hubbard, 1989; de Mestre, 1990) discuss optimal release angle, the angle between the initial velocity vector and horizontal. Lichtenberg and Wills, neglecting any possible relationship between release angle and release speed, proved that the deviation of the “optimal” release angle from the obvious value of 45\textdegree is due to the fact that the release height is approximately 2 m higher than the level of impact. They calculated the optimal release angle, for a speed producing roughly world record distances of 73 ft, to be $\theta_{\text{opt}} = 42.3$\textdegree, and showed that aerodynamic drag has a negligible (<0.1\textdegree) effect on optimal release angle. Curiously, however, there is a large discrepancy between measured release angles (McCoy et al., 1984) and the optimal release angles predicted on their theoretical basis. Film studies by Cureton (1939), Dessureault (1976), Tsirakos et al. (1995), and McCoy et al. indicate that elite shot putters release the shot at an angle to the horizontal considerably less than 42\textdegree, and sometimes as low as 29\textdegree.

All previous developments derive the formula for the optimum angle of release assuming independence of the release angle, release height and release speed, i.e. that there are no constraints among these variables. This assumption of independent velocity does not seem to be physically realistic, however, since the effort needed to project the shot horizontally appears to be much less than that needed to project it vertically (e.g. see the explanation by Zatsiorsky, 1990).

Two previous investigations found similar constraint relations between release variables in javelin throwing, Red and Zogaib (1977) using a 1.14 kg ball of similar mass and Vitasalo and Korjus (1988) using a javelin. Both determined that the release speed decreased

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linearly with release angle, but did not consider other variables or higher-order models. The experiments of Red and Zogaib (1977) were apparently motivated by incorrect theoretical predictions of optimal javelin release angles made by Soong (1975), assuming independence of javelin release speed and angle. Recently Maheras (1995) has undertaken a similar study in the shot put, which developed regression equations for release angle as a function of release velocity and height. In addition there has been brief speculation on the possibility of a similar effect by Tsirakos et al. (1995).

In a review of the throws in track and field, Hubbard (1989) noted that, in all throwing events, “optimal release conditions (for variables) other than velocity depend crucially on how the maximum achievable release velocity is functionally related to the other release conditions”, and this is commented on further in Hubbard (2000). The fundamental hypothesis of this paper is that there is indeed a dependence of achievable release velocity and release horizontal distance on the other release conditions. This is important because this functional dependence may account for the remaining discrepancy between observed and predicted optimal release angles. In addition, in order to train throwers to throw optimally, it is necessary to be able to compute what their desired optimal behavior is. The present paper addresses only the determination of the constraint relations among the four variables; release speed, horizontal distance, release angle and height.

2. Methods

Before we consider constraints among the release variables, we first examine flight. Consider a shot of mass \( m \) released at a height \( h \) with speed \( v \) and angle \( \theta \) between the velocity vector and horizontal. Choose the point on the throwing circle from which the measurement of range is made as the origin of a two-dimensional (2-D) coordinate system with \( x' \) horizontal in the actual throwing direction and \( y' \) vertical (Fig. 1), so the release point has coordinates \((d, h)\).

Numerous previous theoretical studies (e.g. Dyson, 1977; Lichtenberg and Wills, 1978; Townend, 1984; de Mestre, 1990) contain perhaps the most fundamental summary of the flight range, assuming a drag-free model. Only Dyson (1977) and Tabor (1990) included a correction \( d \) because the shot is not necessarily released directly above the edge of the throwing circle. Adding flight range to \( d \) yields

\[
R = d + v^2 \cos \theta \sin \theta + \left( \sin^2 \theta + 2gh/v^2 \right)^{1/2}/g. \tag{1}
\]

Even for world-class shot putters, including air-drag results in less than 1% correction to the range from Eq. (1). Therefore, we here neglect aerodynamic drag and its effect on range.

If \( v, h \) and \( d \) are independent of release angle, the optimal release angle \( \theta_m \) maximizing range (Lichtenberg and Wills, 1978; de Mestre, 1990; Hubbard, 2000) is derived by requiring \( dR/d\theta = 0 \), yielding

\[
\theta_m = \arctan[v/(v^2 + 2gh)^{1/2}]. \tag{2}
\]

However, where \( v, \theta, h \) and \( d \) are not independent, Eq. (2) does not apply and their interdependence must be determined before optimal release conditions can be calculated (Hubbard, 1989). The purpose of this paper is to investigate these relationships.

Two skilled shot putters from the UCD track-and-field team threw the same shot in 36 trials. One (AJB – 106 kg, 1.83 m tall — 22 throws) was in the top 12 nationally and the other (JST – 115 kg, 1.91 m tall — 14 throws) was at college level. Each was asked to produce three sets of relatively high initial angle, normal and relatively low initial-angle throws, respectively, all at maximum effort, and each used the standard rotary throwing technique.

A 200 Hz video (ExpertVision, Santa Rosa, California) motion analysis system was used to determine initial release angle, speed, height and horizontal distance for each throw as described below. A straight desired throwing line (the \( x' \)-axis of a second coordinate system; see Fig. 2) was laid out on the ground bisecting the throwing circle with origin also at the center of the inner board edge. The single camera was levelled and placed at a distance \( c_0 \) (approximately 2.5 m) from, and with its axis perpendicular to, this line so that its field of view encompassed release and the beginning of flight (Fig. 2). Initial 2-D calibration of the camera relied on a calibration object in the \( xy \) plane. Thowers were instructed to throw so that both the release position and initial velocity vector lay in this plane. Even with assistance and feedback, however, the throwers found it very difficult to (and never did) release the shot exactly in the desired plane or in the correct direction. The implications of this on scaling are discussed further below.

Camera data were first processed, using the (naive) 2-D assumption that the throw occurred in the \( xy \) plane.
Pixel coordinates were scaled by the constant initial calibration scale factor and shifted to account for the position of the origin. Typical trajectory segments contained 15–20 co-ordinates of the centroid of the circular image over roughly 1-m of flight at 0.005 s intervals. Impact coordinates \((x_f, z_f)\) were also measured, allowing calculation of actual range.

Parabolic (drag-free) approximations of the trajectory were fitted to the data to yield estimates for \(x\) and \(y\) positions and velocities and compared to the measured range \(R_m\). Results for (JST) are shown in Fig. 3 (as open squares). Clearly there is a substantial discrepancy. A least-squares regression line containing the origin gives \(R_p = 0.963 R_m\), with nearly 4\% average error and substantial maximal deviations of about 10\%, showing that the 2-D approximation and the calculated release conditions based on it were not valid.

The video data appeared very clean, however, and measurement of landing position and range was reasonably accurate (~1 cm). Thus, an alternate scaling method using this data was developed to produce more accurate estimates for release conditions. The basic idea is to include parameters in the model which account for the major error sources due to scaling, and to use video and range data to estimate release conditions and error sources simultaneously, thus removing their effects from the calculated release conditions. This technique is an example of using accurate non-image-related information to correct out-of-plane errors (Sih et al., 2001).

Consider Fig. 2. Initial calibration determines a nominal scale factor \(s_0\) (meters per pixel), which applies only to motion in the calibration plane. However, the general throw was neither released in the desired \(xy\) plane, nor was its \(x'y'\) flight plane parallel to it. Therefore, the correct scale factor \(s\) varies with distance \(c\) from the camera throughout flight according to

\[
s = s_0/c_0 = (c_0 + z_0 + v_x t \tan \phi) s_0/c_0, \tag{3}\]

where \(v_x\) is the \(x\) velocity and \(t\) is the time since release. Combining parameters yields

\[
s = s_1 + k t, \tag{4}\]

where \(s_1\) and \(k\) are interpreted as initial scale factor at release and (constant) time rate of change of scale factor throughout flight, respectively.

Since we propose to scale the pixel data in a somewhat fluid way, it is important to insure that the results remain reasonable. We define a performance index \(J\) that includes two well-known requirements. First, predicted range \(R_p\) using release conditions determined from the scaled data should accurately match the measured range \(R_m\). Second, neglecting aerodynamic drag, the \(y\) component of scaled position should be quadratic in time with its \(r^2\) coefficient \(g a/2\) nearly half the known local value of \(g = 9.7935 \text{ m/s}^2\). The performance index \(J\) is written as

\[
J = (R_m - R_p)^2 + (g a - g)^2. \tag{5}\]

The unknown scaling parameters \(s_1\) and \(k\) are equivalent to the two parameters \(c_0\) and \(z_0\) since

\[
s_1 = (1 + z_0/c_0) s_0 \tag{6}\]

and

\[
k = v_x s_0 \tan \phi/c_0, \tag{7}\]

in which \(v_x\) is nearly constant and \(\phi = \tan^{-1}((z_f - z_0)/(x_f - x_0))\) and where \((x_0, z_0)\) and \((x_f, z_f)\) are the initial and final (impact) positions, respectively. Scaling parameters \(s_1\) and \(k\) were determined iteratively for each throw as follows.
Algorithm for determination of optimal scaling parameters:

1. Make initial guesses for \( s_1 \) and \( k \) (\( s_1 = s_0 \) and \( k = 0 \)).
2. Scale camera data with \( s_1 \) and \( k \) using Eq. (4) to yield 3-D positions versus time.
3. Determine position and velocity at the first frame by fitting a quadratic (drag-free) polynomial in time to this position data. Estimate apparent gravity \( g_a \) as half the quadratic coefficient of the \( y \) regression.
4. Use these initial conditions and Eq. (1) to predict impact point and range.
5. Calculate the performance index \( J \) from Eq. (5) and its gradient with respect to \( s_1 \) and \( k \).
6. Use the nonlinear least-squares estimation technique described in Hubbard and Alaways (1989) to calculate perturbations \( \delta s_1 \) and \( \delta k \) to the scaling parameters \( s_1 \) and \( k \) that decrease the performance index.
7. Stop if corrections \( \delta s_1 \) and \( \delta k \) or the change in \( J \) are too small; otherwise go to step 2.

Note that in step 4, the predicted range is obtained from a drag-free parabolic approximation. Lichtenberg and Wills (1978) showed that, even with velocities near 15 \( \text{m/s} \) and flight times near 2 \( \text{s} \), aerodynamic forces decrease range by only 0.8%. Drag-free approximations applied to very short trajectory segments (\( \Delta t = 0.1 \text{s} \)) incur much smaller errors (0.03%). Because we are interested in the sensitivity of release conditions, and not the conditions themselves, the drag-free approximation was adequate even in the full trajectory.

The optimal scaling technique was implemented using MATLAB. The algorithm converges quadratically and achieves an rms error less than 0.01 \( \text{m} \) between predicted and measured ranges (solid symbols in Fig. 3). It completely resolves the problem from the native 2-D scaling shown by the open square symbols and also achieves agreement between apparent and known values of gravity to better than 0.1%.

Release angle \( \theta \) was calculated from

\[
\theta = \arctan\left(\frac{v_h}{v_h}ight) \tag{8}
\]

and initial speed \( v \) from

\[
v = \left( v_h^2 + v_y^2 \right)^{1/2}, \tag{9}
\]

where the horizontal velocity is given by

\[
v_h = \left( v_x^2 + v_y^2 \right)^{1/2} \tag{10}
\]

and \( v_h \) and \( v_y \) were determined as in step 3 of the algorithm. The two variables \( d \) and \( h \) are the coordinates in the \( x'y' \) plane at release.

Possible constraint relationships among the four variables \( v, d, \theta, \) and \( h \) are the subject of the remainder of the paper. Because of the quadratic dependence of \( R \) on \( v \), it is more sensitive to changes in \( v \) than to changes in the other two variables \( \theta \) and \( h \). For this reason we choose \( v \) as a dependent variable and \( \theta \) and \( h \) as independent variables in one of the constraint relations. Consequently, the remaining variable \( d \) is also chosen as a dependent variable in \( \theta \) and \( h \). Labelling certain variables as independent and others as dependent is somewhat arbitrary, since any constraint relationships among the variables treats them all in an equal footing.

Thus, the terms independent and dependent apply more to the regression model than to the constraint relationships themselves, and should not imply that some variables are more important or fundamental than others.

Multivariate polynomial regression analyses were used to elucidate the relationships imbedded in the release data. The regression equations were solved using a MATLAB program. So that the sensitivities obtained would be as meaningful as possible to the reader, the independent variables in the regressions were chosen as the deviations \( \Delta \theta \) and \( \Delta h \) from mean values \( \theta_m \) and \( h_m \) for each athlete. Quadratic multiple regressions were performed for each thrower of the form

\[
v = C + \Delta \theta \Delta \theta + C_h \Delta h + C_{\theta h} \Delta \theta \Delta h + C_{hh} \Delta h^2 \tag{11}
\]

with a similar equation for the other dependent variable \( d \).

Finally the regression coefficients were tested for statistical significance. The most common null hypothesis \( H_0 \) is that all coefficients except the mean \( C \) in a given regression are zero, i.e. that the dependent variable is not related to the independent ones at all (Bethea and Rhinehart, 1991). This hypothesis is rejected using a one-sided \( F \) test based on the ratio of the mean square due to regression with \( p \) degrees of freedom (MSR) to the mean-squared residual error (MSE). The null hypothesis can be rejected at the 1-\( \alpha \) level if the statistic \( F = \text{MSR}/\text{MSE} > F_{p,(n-p-1),1-\alpha} \) (Bethea and Rhinehart, 1991, p. 171). In the models above, the number of regression degrees of freedom \( p = 5 \).

3. Results

Achievable release speed decreases with increasing release angle at about 1.7 (m/s)/rad and decreases with increasing release height at about 0.8 (m/s)/m, with only small differences in sensitivities between throwers (see top half of Table 1). For each thrower the sensitivities of release speed to \( \theta \) and \( h \) are within 3% of \(-1.68 \) (m/s)/rad (speed decreases as angle increases) and within 15% of \(-0.8 \) (m/s)/m (speed decreases as height increases), respectively. In addition the principal curvatures of the speed–angle–height constraints for the two athletes are similar in sign and within a factor of two in magnitude. In addition to the regression coefficients, Table 1
contains the means of the independent and dependent variables, and the rms errors of the fits.

Horizontal release distance also decreases with increasing release angle at about 1.7 m/rad but increases with increasing release height at about 1.3 m/m, again with only small differences between the two throwers (bottom half of Table 1). Although mean values for horizontal release distance for both throwers were negative (each usually released behind the line) their magnitudes were less than 0.1 m. There are significant sensitivities of \( d \) to both \( \theta \) and \( h \), however, which are again nearly equal for each thrower. As with the release speed sensitivities, there are only relatively small differences between the two throwers.

The constraints are perhaps more easily visualized graphically. The coefficients from Table 1 were used in Eq. (11) to produce each thrower’s constraint surfaces for \( v = v(\theta, h) \) and \( d = d(\theta, h) \). These are illustrated in Figs. 4 and 5, respectively, using the same scales for \( v, d, \theta \) and \( h \). Also, included are the experimental points from which the least-squares fits and coefficients were calculated. A measure of the errors between the data and the fits are depicted by vertical lines connecting the data to the surfaces.

The regression coefficients are shown to be statistically significant by the last column of Table 1. This contains the confidence level \( 1 - \alpha \) at which the null hypothesis \( H_0 \), that the dependent variable for that row is not related to the independent ones at all, is rejected. In every case \( 1 - \alpha > 0.95 \). The least statistically significant relationship is that for \( v \) for AJB but all others have statistical significance greater than 0.995.

### 4. Discussion

These constraints are inevitably caused by limitations of the geometry and musculoskeletal structure and ability of the thrower. Because different throwers have different heights, strengths, arm lengths and muscular dynamic function (force–velocity and force–length characteristics), one might expect that such constraint relationships would be complex and different for different throwers. Although we are not able to draw any universal conclusions based only on two throwers, it is interesting that the constraints for the two so physically dissimilar athletes are so nearly equal.

The fits to \( v \) and \( d \) in Figs. 4 and 5 are relatively good in the sense that the rms errors are only about 0.13 m/s and 0.04 m, respectively, roughly 10% of the variations of \( v \) and \( d \) in the experiments.

Maheras (1995) developed regression equations for shot put release angle as a function of velocity and height, and thus his results are difficult to compare with ours. The negative slopes of his velocity-release angle scattergrams, however, range from \(-2.8\) to \(-5.7\) (m/s)/rad, roughly a factor of two higher than the results in this study, \(-1.68\) (m/s)/rad.

The release velocity sensitivity to release angle in the shot put of \(-1.68\) (m/s)/rad is about 4 times smaller than the similar javelin sensitivity of \(-7.28\) (m/s)/rad (Red and Zogaib, 1977, Viitasalo and Korjus, 1988), but the negative signs indicate that in both cases the throwers can throw faster at smaller release angles.

The magnitudes of these sensitivities of \( d \) to \( \theta \) and \( h \) are reasonable based on geometric arguments. Because thrower AJB is 0.08 m shorter than JST and had correspondingly shorter arms, his mean release heights were considerably less (0.16 m) than those of JST. Although the derivative of release \( d \) with respect to release \( h \) is not the same concept as the variation of horizontal distance with height along a trajectory \( (dx/dy) = \cot \theta \), their magnitudes should be of the same order. For the overall mean release angle \( \theta = 0.68 \) rad, \( \cot \theta = 1.23 \), which is remarkably close to 1.2, the mean sensitivity of \( d \) to \( h \) from Table 1.

The constraint relations discussed above can be interpreted as approximate limitations on the ability of a thrower to achieve combinations of release speed and horizontal release distance which will increase attainable range. This is because the experiments were conducted with a request for maximal effort by the throwers. Due to the nature of the regression process, the constraint surfaces roughly bisect the data points. The true limiting surfaces (actual boundaries of performance) will be nearly parallel to these but shifted slightly to lie outside
all (or possibly on some of) the data. As an approximation we may take the constraint surfaces determined above as adequate representations of the true limiting constraint surfaces.

The limitations of the approach we have followed to identify the constraints come from two main factors that prevent their exact determination. Most importantly we have assumed that the throwers were operating at “maximal effort”. But the throwers are unable to behave like machines and thus are not likely to achieve repeatedly their maximal velocity in every throw, although they might have been trying as hard as possible to do so. Were they able to, all the data points would lie exactly on the actual constraint surfaces. Secondly, even if they were able to do this, ubiquitous measurement errors would prevent precise determination of the release conditions and hence the constraints.

Whatever the constraints are, they will reduce the problem of finding the release conditions for maximum range to that of optimizing a function of only two independent variables. This is considerably more complicated than the analyses undertaken by most researchers to date (Maheras, 1995). Previous efforts, except Tabor (1990) who assumed that \( d \) was constant, have postulated no relationships among \( v \), \( d \), \( h \), and \( \theta \). They merely sought the maximum range for fixed values of \( v \) and \( h \) with only \( \theta \) varying and assuming \( d = 0 \) or another constant.

The introduction of the horizontal release distance \( d \) in the total range expression for the throw of a shot, and the recognition that both \( d \) and the release speed \( v \) depend on the other release conditions, \( \theta \) and \( h \), provide a framework for a more accurate optimization of the throw to achieve maximum distance. This optimization depends crucially on the knowledge of the constraints relating the release variables for each athlete. This paper has described experiments, data processing techniques, and multivariate regression analyses used to develop these constraints.

In summary, for a given thrower, the shot can be thrown faster when thrown at smaller release angles and when released lower. Achievable release speed decreases with increasing release angle at about 1.7 (m/s)/rad and decreases with increasing release height at about 0.8 (m/s)/m. Release horizontal distance decreases with increasing release angle at about 1.7 m/rad, but increases with increasing release height at about 1.3 m/m. These sensitivities can be used in optimization studies to...
predict optimal release conditions which may agree more favorably with experiment than in the past.

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References


