Investigation of the kinetics of anaerobic metabolism by analysis of the performance of elite sprinters

A.J. Ward-Smith*, P.F. Radford
Department of Sport Sciences, Brunel University, Osterley Campus, Borough Road, Isleworth, Middlesex TW7 5DU, UK
Accepted 2 February 2000

Abstract

The principal motivation for the present work was the study of the kinetics of anaerobic metabolism. A new mathematical model of the bioenergetics of sprinting, incorporating a three-equation representation of anaerobic metabolism, is developed. Results computed using the model are compared with measured data from the mens’ finals of the 100 m event at the 1987 World Championships. The computed results closely predict the overall average performance of the competitors over the course of the entire race. Further calculations show the three-equation model of anaerobic metabolism to be a significant improvement over the previous one-equation model. Representative values of time constants that govern the rate of anaerobic energy release have been determined for elite male athletes. For phosphocreatine utilisation, values for $\lambda_2 = 0.20$ s$^{-1}$ and $\psi_2 = 3.0$ s$^{-1}$ are consistent with data previously reported in the literature. New values of $\lambda_3 = 0.033$ s$^{-1}$ and $\psi_3 = 0.34$ s$^{-1}$ are proposed as offering an improved representation of the kinetics of oxygen-independent glycolysis. For the first time, tentative values for the time constants of ATP utilisation, $\lambda_1 = 0.9$ s$^{-1}$ and $\psi_1 = 20$ s$^{-1}$, are suggested. The maximum powers developed during sprinting by oxygen-independent glycolysis, PCr utilisation and endogenous ATP utilisation were calculated as 34.1, 30.1 and 16.6 W kg$^{-1}$, respectively, with an overall maximum anaerobic power of 51.6 W kg$^{-1}$. Sample calculations show the mathematical model can be used in principle to derive data on the kinetics of anaerobic metabolism of individual athletes. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Running; Sprinting; Biomechanics; Bioenergetics; Anaerobic; Glycolysis; Metabolism

1. Introduction

Many factors affect sprinting performance (Radford, 1990), but one factor is particularly important in the present context. When sprinting is initiated, the ability to move rapidly from the starting blocks depends critically on the availability of energy through anaerobic metabolism. Between 92 and 93% of the chemical energy converted during a 100 m sprint is from anaerobic sources (Ward-Smith, 1985; Peronnet and Thibault, 1989). In a recent theoretical contribution to the understanding of anaerobic metabolism, Ward-Smith (1999b) derived three mathematical equations, describing the power relationships for oxygen-independent glycolysis, phosphocreatine (PCr) utilisation and the utilisation of endogenous ATP. The mathematical relations for oxygen-independent glycolysis and phosphocreatine utilisation were shown to be consistent with the broad trends of experimental results collected by Sipri (1995) but, because the experimental data displayed very considerable scatter, further work is required to investigate the kinetics of anaerobic metabolism.

An understanding of the relationship between running records and physiological data has a distinguished history, following the pioneering work of Hill (1925). Several mathematical models of the bioenergetics of running have been published (Lloyd, 1967; Ward-Smith, 1985; Peronnet and Thibault, 1989; Di Prampero et al., 1993; Ward-Smith and Mobey, 1995). These mathematical methods relate running performance to physiological parameters by setting down the individual contributions to the whole-body energy balance during running. They incorporate a single-equation model of anaerobic metabolism and have been used to predict overall running time. The distance-time history of elite sprinters measured at regular intervals over 100 m have been reported by Moravec et al. (1988).

The accurate measurement of anaerobic power during vigorous exercise is difficult, and the provision of new
insights into the kinetics of anaerobic metabolism using a theoretical approach was the motivation for the present work. This paper has several objectives. The initial objective is to develop a new mathematical model of sprinting, incorporating the three-equation model of anaerobic metabolism as well as several other new features. The second objective is to obtain new scientific evidence for the validity or otherwise of the three-equation model of anaerobic metabolism, by comparing the distance-time measurements of Moravec et al. (1988) with corresponding theoretical predictions. Such detailed comparisons are a much sterner test of the efficacy of a mathematical model than the prediction of overall running time, and have not been published before. If the predictions and track measurements are shown to be consistent, then the three-equation model is supported; the converse applies equally. Third, if the comparisons of sprinting performance are favourable, the aim is to derive new or improved data on the parameters defining the kinetics of anaerobic metabolism. These data will be subject to a sensitivity analysis, predictions using the three-equation model of anaerobic metabolism will be compared with calculations obtained from the one-equation model, and the applicability of the mathematical model to the performance of individual athletes will be investigated.

2. Method

The energy \( C \) released by chemical conversion at the muscles passes through a number of intermediate stages and, in conformity with the first law of thermodynamics, is ultimately transformed into external work, \( W \), expended on the centre-of-mass of the runner, or mechanical energy degraded into heat, \( H \). The mathematical model is simplified by considering the broad changes of energy level, and ignoring the cyclical variations associated with the stride pattern.

The power equation for running, expressed per unit body mass, can be written in the form

\[
\frac{dC}{dt} = \frac{dH}{dt} + \frac{dW}{dt}, \tag{1}
\]

where the left-hand side represents the rate of chemical energy conversion, and the first and second terms on the right-hand side are, respectively, the rate of degradation of mechanical energy into thermal energy and the rate of external mechanical working. The power contributions are expressed relative to the basal metabolic rate.

In current mathematical models of the bioenergetics of sprinting (Lloyd, 1967; Ward-Smith, 1985; Ward-Smith, 1999a; Ingen Schenau et al., 1991) it is customary to consider the contributions of two principal components to the rate of external working: the rate of increase of the kinetic energy of the centre-of-mass of the runner in the horizontal direction, \( v \frac{dv}{dt} \), and the rate of working against aerodynamic drag, \( \frac{dv}{m} \). Here \( m \) is body mass, \( v \) is the velocity of the athlete over the ground, and \( D \) is aerodynamic drag. For improved accuracy, a third component must be taken into consideration. After leaving the starting blocks, the distance of the athlete’s centre-of-mass above the ground, \( h_c \), increases significantly, so the rate of working against gravity of the body’s centre-of-mass is given by \( gh_c \frac{dh_c}{dt} \), where \( g \) is the gravitational acceleration. Therefore, the rate of external mechanical working (per unit body mass) is

\[
\frac{dW}{dt} = \frac{Dv}{m} + gh_c \frac{dh_c}{dt} + \frac{dv}{dt}. \tag{2}
\]

The drag \( D \) is given by

\[
D = \frac{1}{2} \rho (v - V_w)^2 SC_D = \frac{1}{2} \rho (v - V_w)^2 A_D \tag{3}
\]

where \( \rho \) is the air density, \( V_w \) is the wind speed (an assisting wind being taken as positive), \( S \) is the projected frontal area of the runner, \( C_D \) is the drag coefficient and \( A_D = SC_D \) is the drag area.

Following the method of Ward-Smith (1999a), the rate of degradation of mechanical energy is

\[
\frac{dH}{dt} = Av, \tag{4}
\]

where \( A \) is the rate of degradation of mechanical energy into thermal energy per unit velocity and can be written as

\[
A = A_0 + A_1 V_w, \tag{5}
\]

where \( A_0 \) and \( A_1 \) are constants.

The rate of chemical energy conversion is given by

\[
\frac{dC}{dt} = \frac{dC_{an}}{dt} + \frac{dC_{aer}}{dt}, \tag{6}
\]

where the first and second terms on the right-hand side are, respectively, the contributions from anaerobic and aerobic metabolism.

Eqs. (1)–(6) combine to yield

\[
\frac{dC_{an}}{dt} + \frac{dC_{aer}}{dt} = Av + K(v - V_w)^2 + gh_c \frac{dh_c}{dt} + \frac{dv}{dt} \tag{7}
\]

where

\[
K = \frac{\rho SC_D}{2m}. \tag{8}
\]

Based on the experimental work reported by Margaria (1976) and others, it is well established that the early stages of aerobic energy release can be written in the form

\[
\frac{dC_{aer}}{dt} = R[1 - \exp(-\lambda t)], \tag{9}
\]
where $R$ is the maximum sustainable aerobic power and $\lambda$ is a parameter governing the rate of aerobic energy release.

Substitution of Eq. (9) in Eq. (7) and rearrangement lead to

$$\frac{dC_{an}}{dt} = Av + K \epsilon (v - V_w)^2 + g \frac{dh}{dt} + \frac{dv}{dt}$$

$$- R[1 - \exp(-\lambda v t)]$$

(10)

so that the kinetics of anaerobic metabolism can be quantified if the right-hand side of Eq. (10) can be evaluated.

2.1. The kinetics of anaerobic metabolism

In existing mathematical models of the bioenergetics of sprinting (Lloyd, 1967; Ward-Smith, 1985, 1999a; Ingen Schenau et al., 1991), the anaerobic power under conditions of maximum exertion are represented by

$$\frac{dC_{an}}{dt} = P_{max} \exp(-\lambda_{an} t),$$

(11)

where $P_{max}$ represents the maximum anaerobic power and $\lambda_{an}$ is a parameter governing the rate of aerobic energy release. Although this relation best describes the period when oxygen-independent glycolysis prevails, it has been used (Lloyd, 1967; Ward-Smith, 1985, 1999a; Ingen Schenau et al., 1991) as an approximation for all $t > 0$. Incorporating ideas advanced by Margaria (1976) — that a close relationship exists between the build-up of aerobic power and the decline of anaerobic power — Ward-Smith (1985) argued that the same value of $\lambda$ could be used in Eqs. (9) and (11). Whilst Eq. (9) is now well established as a result of extensive measurements in the laboratory, Eq. (11) is less securely based. The successful application of the single anaerobic equation to predict overall running time can be ascribed to two self-compensating features. Effectively, it overestimates the glycolytic contribution in the first few seconds of exercise, but in compensation fails to include the ATP and PCR contributions.

Here we incorporate the fuller mathematical description of anaerobic metabolism applicable to small values of $t$ proposed by Ward-Smith (1999b). We use the suffix notation, with $n = 1, 2, 3$, where 1 relates to ATP utilisation, 2 to phosphocreatine utilisation and 3 to oxygen-independent glycolysis. The power equations are of the form

$$P_n = (P_{max})_n (r_n)^{(1/r_n)} [(1 + r_n)/(r_n)]^{(1 + r_n)/(r_n)}$$

$$[1 - \exp(-\psi_n t)] \exp(-\lambda_n t).$$

(12)

In Eq. (12), $P_n$ and $(P_{max})_n$ represent the instantaneous and maximum powers of the $n$th component, respectively, $\psi_n$ and $\lambda_n$ are time-constants relating to the $n$th component, and

$$r_n = \frac{\psi_n}{\lambda_n}.$$  

(13)

Ward-Smith (1999b) proposed that $\lambda_3 = \lambda$.

Thus the anaerobic power is given by

$$\frac{dC_{an}}{dt} = \sum_{n=1,2,3} (P_{max})_n (r_n)^{(1/r_n)} [(1 + r_n)/(r_n)]^{(1 + r_n)/(r_n)}$$

$$[1 - \exp(-\psi_n t)] \exp(-\lambda_n t).$$

(14)

2.2. Application to sprinting

Comprehensive data were collected (Moravec et al., 1988) during both the heats and finals of the mens’ 100 m event at the 1987 World Championships, held in Rome. Measurements were made at 10 m intervals along the track. These results showed that the leading athletes were running at their fastest, and hence with maximum effort, during the final, and so the data from that race are used for comparison with data computed from the mathematical model set out above.

Eq. (10) can be rewritten as

$$\frac{dv}{dt} = \frac{1}{v}[Av + K \epsilon (v - V_w)^2 + g \frac{dh}{dt} - \frac{dC_{an}}{dt} - R(1 - \exp(-\lambda v t))].$$

(15)

Also

$$\frac{dx}{dt} = v.$$  

(16)

During running the nominal angle of the athlete’s torso relative to the horizontal, $\theta$, varies. Following Ward-Smith (1999a), $\theta$ is given by the relation

$$\tan \theta = \frac{mg}{(mdv/dt + D)} = \frac{g}{dv/dt + K(v - V_w)^2}.$$

(17)

We define

$$K_0 = \frac{\rho D C_D}{2m}$$

(18)

so that $K$ and $K_0$ are related by

$$K = K_0 \sin \theta.$$  

(19)

Baumann (1976) has shown that, for a typical athlete, the centre-of-mass is raised from its initial position, $h_0$, of 0.65 m in the blocks to about 1.0 m after some 5 m have been run. In the present analysis the height of the centre-of-mass above the horizontal running surface, $h_c$, was computed from

$$h_c = h_{cm} \sin \theta,$$

(20)
where \( h_{cm} \) represents the height of the centre-of-mass above the ground when the athlete is standing vertically. The increase in potential energy (per unit body mass) measured relative to the condition in the starting block, \( E_h \), is given by

\[
E_h = g(h_e - h_0),
\]

where \( h_0 = 0.65 \text{ m} \). For small values of \( t \), some of the computed values of \( h_e \) fell in the range \( h_e < 0.65 \text{ m} \); for these values \( E_h \) was taken as zero.

When an athlete adopts the “set” position in the starting blocks, his centre-of-mass is a distance \( h_b \) behind the starting line, typically about 0.16–0.19 m (Baumann, 1976; Mero et al., 1983). Here a constant value for \( h_b \) of 0.17 m has been used for all athletes. Eqs. (15) and (16) were solved by numerical integration using a fourth-order Runge–Kutta method, with a time-step of 0.01 s and between the limits \( t = 0, x = -h_b \) and \( t = T, x = 100 \text{ m} \).

Integration of Eq. (7) between the limits \( t = 0 \) and \( t = T \) yields, after rearrangement

\[
v = \sqrt{\left[2(Can + C_{aer} - Ax - \{Kt(v - V_w)^2 dt - g(h_e - h_0)\})\right].}
\]

(22)

Most of the terms in the power equation are analytical, and therefore they can be integrated exactly. To take advantage of the analytical forms of \( C_{aer} \), \( Can \) and \( Eh \) and to minimise the growth of the residual error during numerical integration, after each second iteration the value of \( v \) was recalculated to provide an exact energy balance using Eq. (22). The aerodynamic drag term was integrated using Simpson’s Rule and \( C_{aer} \) and \( Can \) were obtained by integrating Eqs. (9) and (14), respectively, to yield

\[
C_{aer} = RT - R[1 - \exp(-\lambda T)]/\lambda
\]

and

\[
Can = \sum_{n=1,2,3} (Can)_n h_n,
\]

(24)

where

\[
(Can)_n = (P_{max} h)(r_n)^{1/r_n} \left[(1 + r_n)(1 + r_n)^{1/r_n} \frac{\psi_n}{\lambda_n(\lambda_n + \psi_n)} \right. \\
\left. - \exp(-\lambda_n T)^{\psi_n} + \lambda_n[1 - \exp(-\psi_n T)]]/\lambda_n(\lambda_n + \psi_n).
\]

(25)

The following numerical values were used for computing running performance \( R = 25 \text{ W kg}^{-1}; V_w = 0.95 \text{ m s}^{-1}; A = 3.96 \text{ kg}^{-1} \text{ m}^{-1}; K_0 = 0.0029 \text{ m}^{-1}; h_{cm} = 1 \text{ m}. \)

There is a short delay of duration \( t_d \) from the moment at which the starting gun and the measuring equipment are simultaneously activated and the time at which the sprinter reacts to the starting signal (Radford, 1990). Hence, the total running time, \( t_r \), and the actual running time, \( t \), are related by

\[
t_r = t + t_d.
\]

The reaction times used here are the official times, based on the Seiko timing equipment, as reported by Moravec et al. (1988).

The results for the first seven competitors to finish in the final of the 100 m sprint in the 1987 World Championships were used in the analysis. The eighth finalist, who finished in a time of 16.23 s, was suffering from an injury, and so no attempt was made to incorporate his performance. The overall average performance of the first seven athletes was evaluated. Actual and computed running times were compared at 10 m intervals along the track. In the main analysis, anaerobic metabolism was modelled using Eq. (14). To gain initial experience with the analysis the following tentative values proposed by Ward-Smith (1999b) were used: \( \lambda_3 = 0.049 \text{ s}^{-1} \), \( \psi_3 = 0.53 \text{ s}^{-1} \), \( \lambda_2 = 0.20 \text{ s}^{-1} \) and \( \psi_2 = 3.0 \text{ s}^{-1} \). Values of \( \{P_{max}, \lambda_n, \psi_n\} \) for \( n = 1, 2, 3 \) were varied systematically by small increments, running times calculated from the present mathematical model were compared with the track data, and the parameter values which provided the lowest overall rms error value were selected.

For comparison with the main set of calculations, the overall average performance was also computed with the anaerobic metabolism described by the one-equation model, replacing Eq. (14) by Eq. (11).

3. Results

A new mathematical model of sprinting, in which the three-equation model of anaerobic metabolism is embedded, has been successfully developed. Additional refinements included in the analysis are: the contribution of the vertical movement of the centre-of-mass to the overall energy balance, allowances for the initial position of the centre-of-mass relative to the starting line, and the effect of reaction time on overall running time.

The overall group average performance of the male athletes who contested the 1987 World Championship 100 m final has been computed using the new mathematical model of sprinting. Computed running times are in close agreement with the measured running times throughout the course of the entire race (Table 1), confirming that the three-equation model provides an excellent description of the kinetics of anaerobic metabolism. There is a very low rms error of less than 0.01 s, and the overall running time is predicted within 0.01 s.

The analysis yields values for the time constants \( \lambda_n \) and \( \psi_n \), used in conjunction with Eq. (12) and applicable to elite male athletes exercising at high-intensity levels. For oxygen-independent glycolysis, values of \( \lambda_3 = 0.033 \text{ s}^{-1} \) and \( \psi_3 = 0.34 \text{ s}^{-1} \) were obtained. The analysis yielded \( \lambda_2 = 0.20 \text{ s}^{-1} \) and \( \psi_2 = 3.0 \text{ s}^{-1} \) which govern the rate of
ties. Maximum powers developed by oxygen-independent anaerobic metabolism were varied in turn by ±5% from the nominal value. The table shows the effects on computed running time and rms error, and is presented in rank order.

**Table 1**

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Running time (s)</th>
<th>Energy contributions</th>
<th>Parameter Multiplication factor</th>
<th>Computed running time (s)</th>
<th>rms error (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.94</td>
<td>1.93</td>
<td>16.66</td>
<td>10.11</td>
<td>10.12</td>
</tr>
<tr>
<td>20</td>
<td>2.98</td>
<td>2.98</td>
<td>19.47</td>
<td>10.12</td>
<td>10.14</td>
</tr>
<tr>
<td>30</td>
<td>3.94</td>
<td>3.93</td>
<td>20.50</td>
<td>10.13</td>
<td>10.10</td>
</tr>
<tr>
<td>40</td>
<td>4.83</td>
<td>4.84</td>
<td>20.92</td>
<td>10.14</td>
<td>10.10</td>
</tr>
<tr>
<td>50</td>
<td>5.71</td>
<td>5.72</td>
<td>21.11</td>
<td>10.15</td>
<td>10.11</td>
</tr>
<tr>
<td>70</td>
<td>7.47</td>
<td>7.46</td>
<td>21.23</td>
<td>10.17</td>
<td>10.13</td>
</tr>
<tr>
<td>80</td>
<td>8.35</td>
<td>8.34</td>
<td>21.25</td>
<td>10.18</td>
<td>10.14</td>
</tr>
<tr>
<td>90</td>
<td>9.23</td>
<td>9.22</td>
<td>21.25</td>
<td>10.19</td>
<td>10.15</td>
</tr>
<tr>
<td>100</td>
<td>10.11</td>
<td>10.12</td>
<td>21.26</td>
<td>10.20</td>
<td>10.16</td>
</tr>
</tbody>
</table>

*rms error (s) = 0.006

$P_{max,l1} = 34.1 \text{ W kg}^{-1}$, $l_3 = 0.033 \text{ s}^{-1}$, $\psi_1 = 0.34 \text{ s}^{-1}$

$P_{max,l2} = 30.1 \text{ W kg}^{-1}$, $l_2 = 0.20 \text{ s}^{-1}$, $\psi_2 = 3.0 \text{ s}^{-1}$

$P_{max,l3} = 16.6 \text{ W kg}^{-1}$, $l_1 = 0.9 \text{ s}^{-1}$, $\psi_3 = 20 \text{ s}^{-1}$

**Table 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Multiplication factor</th>
<th>Computed running time (s)</th>
<th>rms error (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{max,l1}$ (W kg$^{-1}$)</td>
<td>1.00</td>
<td>10.12</td>
<td>0.01</td>
</tr>
<tr>
<td>$P_{max,l2}$ (W kg$^{-1}$)</td>
<td>1.05</td>
<td>9.93</td>
<td>0.10</td>
</tr>
<tr>
<td>$P_{max,l3}$ (W kg$^{-1}$)</td>
<td>0.95</td>
<td>10.31</td>
<td>0.10</td>
</tr>
<tr>
<td>$l_1$ (s$^{-1}$)</td>
<td>1.05</td>
<td>10.17</td>
<td>0.03</td>
</tr>
<tr>
<td>$l_2$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.06</td>
<td>0.03</td>
</tr>
<tr>
<td>$l_3$ (s$^{-1}$)</td>
<td>1.05</td>
<td>10.08</td>
<td>0.03</td>
</tr>
<tr>
<td>$\psi_1$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.15</td>
<td>0.03</td>
</tr>
<tr>
<td>$\psi_2$ (s$^{-1}$)</td>
<td>1.05</td>
<td>10.08</td>
<td>0.02</td>
</tr>
<tr>
<td>$\psi_3$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.15</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Table 3**

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Running time (s)</th>
<th>Parameter Multiplication factor</th>
<th>Computed running time (s)</th>
<th>rms error (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.94</td>
<td>$P_{max}$ (W kg$^{-1}$)</td>
<td>50.5</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>2.98</td>
<td>$l_1$ (s$^{-1}$)</td>
<td>1.05</td>
<td>10.13</td>
</tr>
<tr>
<td>30</td>
<td>3.94</td>
<td>$l_2$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.09</td>
</tr>
<tr>
<td>40</td>
<td>4.83</td>
<td>$l_3$ (s$^{-1}$)</td>
<td>1.05</td>
<td>10.13</td>
</tr>
<tr>
<td>50</td>
<td>5.71</td>
<td>$\psi_1$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.12</td>
</tr>
<tr>
<td>60</td>
<td>6.59</td>
<td>$\psi_2$ (s$^{-1}$)</td>
<td>1.05</td>
<td>10.10</td>
</tr>
<tr>
<td>70</td>
<td>7.47</td>
<td>$\psi_3$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.11</td>
</tr>
<tr>
<td>80</td>
<td>8.35</td>
<td>$\psi_4$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.11</td>
</tr>
<tr>
<td>90</td>
<td>9.23</td>
<td>$\psi_5$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.11</td>
</tr>
<tr>
<td>100</td>
<td>10.11</td>
<td>$\psi_6$ (s$^{-1}$)</td>
<td>0.95</td>
<td>10.11</td>
</tr>
</tbody>
</table>

*rms error (s) = 0.063

$P_{max} = 50.5 \text{ W kg}^{-1}$

$\lambda_n (\text{s}^{-1}) = 0.01$

PCr utilisation. The energy contribution from endogenous ATP is very small and delivered very rapidly; the values $l_1 = 0.9 \text{ s}^{-1}$ and $\psi_1 = 20 \text{ s}^{-1}$ reflect these general properties. Maximum powers developed by oxygen-independent glycolysis, PCr utilisation and endogenous ATP utilisation were determined as 34.1, 30.1 and 16.6 W kg$^{-1}$, respectively. The times at which these peaks occur were evaluated as 7.1, 0.92 and 0.16 s, respectively. A separate calculation
Table 4
Final of 100 m World Championship, Rome, 1987. Comparison of computed running times (column (a)) with measured running times (column (b)) for the first seven finalists. Anaerobic power calculated using Eq. (14). There was a following wind, $V_w = 0.95 \text{ m s}^{-1}$. For the purposes of computation, the following values were used for all competitors: $R = 25 \text{ W kg}^{-1}$; $A = 3.96 \text{ J kg}^{-1} \text{ m}^{-1}$; $K_0 = 0.0029 \text{ m}^{-1}$; $h_{\text{in}} = 1 \text{ m}$; $h_i = 0.65 \text{ m}$; $h_h = 0.17 \text{ m}$; $\lambda_3 = 0.033 \text{ s}^{-1}$; $\psi_3 = 0.34 \text{ s}^{-1}$; $\lambda_2 = 0.20 \text{ s}^{-1}$; $\psi_2 = 3.0 \text{ s}^{-1}$; $\lambda_1 = 0.9 \text{ s}^{-1}$; $\psi_1 = 20 \text{ s}^{-1}$. Also shown are the derived values of $(P_{\text{max}})_1, (P_{\text{max}})_2, (P_{\text{max}})_3$ for each athlete.

<table>
<thead>
<tr>
<th>Sprinter</th>
<th>B. Johnson</th>
<th>C. Lewis</th>
<th>R. Stewart</th>
<th>L. Christie</th>
<th>A. Kovacs</th>
<th>V. Bryzgin</th>
<th>L. McCrae</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finishing Position</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Reaction time, $t_\delta$ (s)</td>
<td>0.109</td>
<td>0.196</td>
<td>0.235</td>
<td>0.155</td>
<td>0.201</td>
<td>0.139</td>
<td>0.225</td>
</tr>
<tr>
<td>Distance (m)</td>
<td>Running time</td>
<td>Running time</td>
<td>Running time</td>
<td>Running time</td>
<td>Running time</td>
<td>Running time</td>
<td>Running time</td>
</tr>
<tr>
<td></td>
<td>(a) (s)</td>
<td>(b) (s)</td>
<td>(a) (s)</td>
<td>(b) (s)</td>
<td>(a) (s)</td>
<td>(b) (s)</td>
<td>(a) (s)</td>
</tr>
<tr>
<td>10</td>
<td>1.83</td>
<td>1.84</td>
<td>1.93</td>
<td>1.94</td>
<td>1.95</td>
<td>1.97</td>
<td>1.96</td>
</tr>
<tr>
<td>20</td>
<td>2.86</td>
<td>2.86</td>
<td>2.96</td>
<td>2.96</td>
<td>3.01</td>
<td>3.01</td>
<td>3.02</td>
</tr>
<tr>
<td>30</td>
<td>3.79</td>
<td>3.80</td>
<td>3.90</td>
<td>3.91</td>
<td>3.97</td>
<td>3.97</td>
<td>3.98</td>
</tr>
<tr>
<td>40</td>
<td>4.68</td>
<td>4.67</td>
<td>4.78</td>
<td>4.78</td>
<td>4.88</td>
<td>4.87</td>
<td>4.89</td>
</tr>
<tr>
<td>50</td>
<td>5.54</td>
<td>5.53</td>
<td>5.65</td>
<td>5.64</td>
<td>5.76</td>
<td>5.76</td>
<td>5.78</td>
</tr>
<tr>
<td>70</td>
<td>7.24</td>
<td>7.23</td>
<td>7.35</td>
<td>7.36</td>
<td>7.48</td>
<td>7.49</td>
<td>7.50</td>
</tr>
<tr>
<td>80</td>
<td>8.10</td>
<td>8.10</td>
<td>8.21</td>
<td>8.22</td>
<td>8.34</td>
<td>8.35</td>
<td>8.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>rms error (s)</th>
<th>0.008</th>
<th>0.009</th>
<th>0.007</th>
<th>0.010</th>
<th>0.011</th>
<th>0.011</th>
<th>0.009</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(P_{\text{max}})_1$ (W kg$^{-1}$)</td>
<td>34.6</td>
<td>35.2</td>
<td>32.2</td>
<td>34.6</td>
<td>34.0</td>
<td>34.1</td>
<td>32.0</td>
</tr>
<tr>
<td>$(P_{\text{max}})_2$ (W kg$^{-1}$)</td>
<td>32.0</td>
<td>31.0</td>
<td>29.6</td>
<td>29.6</td>
<td>29.1</td>
<td>28.1</td>
<td>28.8</td>
</tr>
<tr>
<td>$(P_{\text{max}})_3$ (W kg$^{-1}$)</td>
<td>16.6</td>
<td>16.6</td>
<td>16.6</td>
<td>13.7</td>
<td>17.0</td>
<td>16.6</td>
<td>23.3</td>
</tr>
</tbody>
</table>
showed that the overall maximum anaerobic power, achieved at $t = 1.4\ \text{s}$, was $5.16\ \text{W kg}^{-1}$.

A sensitivity analysis was made to investigate the effects of systematic changes to each of the anaerobic parameters. One parameter was changed at a time. The effects on predicted running time and rms error of a $\pm 5\%$ change to the nominal value of each parameter was calculated (Table 2). The factors are ranked in descending order, with the most influential factor first. For each component of metabolic power, the dominant factor is $P_{\text{max}1}$, and, broadly, the components rank according to the magnitude of their contribution to the overall energy balance (Table 1). However, there are some variations from this generalisation. For example, the analysis is shown (Table 2) to be relatively sensitive to the value of $\dot{\lambda}_2$.

The overall average performance was next computed using the one-equation model of anaerobic metabolism (Table 3). Values of $P_{\text{max}1}$ and $\dot{\lambda}_2$ were selected to minimise the rms error, and the latter was found to be much greater than the figure obtained when using the three-equation model (Table 1). Also, at both the 10 and 100 m marks, large discrepancies between the predicted and measured running times were found. The one-equation model is shown to overestimate the anaerobic power in the early stages of exercise (Table 3). These comparisons provide strong evidence that the three-equation model is a significant improvement over the one-equation model.

The individual performances of seven athletes have been calculated, showing that the analysis is in principle applicable to individual athletes. Values of $\dot{\lambda}_n$ and $\psi_n$, for $n = 1, 2, 3$, found from the overall average performance (Table 1) were retained, and the rms error was minimised by optimising the values of $(P_{\text{max}1}, P_{\text{max}2}, P_{\text{max}3})$. Good agreement between computed and measured running times has been achieved throughout the course of the entire 100 m race (Table 4). The anaerobic parameter values shown here for each athlete should be regarded as indicative. To obtain more reliable data on the kinetics of anaerobic metabolism for an individual athlete, values of $R, A, K_0, h_1, h_2$ and $h_{\text{im}}$ specific to the athlete are required and a series of several runs should be analysed.

4. Discussion

The results in Table 1 demonstrate that the incorporation of the new three-equation model of anaerobic metabolism into the method for calculating sprinting performance gives very good agreement between computed and measured running times. The mathematical model is able to follow accurately the distance-time history over the full course of the 100 m race. The calculations are within a tolerance band of $\pm 0.02\ \text{s}$ and the rms error of 0.006 s is very small. The results provide strong supporting evidence for the use of the three-equation model of anaerobic metabolism and demonstrate that it is a considerable improvement over the one-equation model. The high level of agreement between calculated and track results, using the three-equation model, deserves some additional words of comment. The mathematical form of the three equations, previously justified on theoretical grounds (Ward-Smith, 1999b), was retained here. Anaerobic metabolism was not described by any arbitrary mathematical expression; nor was agreement achieved simply by adding terms to a series expansion until an acceptable residual error has been obtained. The degree of correlation achieved between the calculated and actual running times — bearing in mind that the numerical values of numerous parameters, such as $R, A, K_0$, etc., were fixed on the basis of prior knowledge and were not allowed to float free — is, in the circumstances, quite extraordinarily good.

Several aspects of the kinetics of anaerobic energy release during vigorous exercise, exemplified here by the performance of elite male sprinters, have been quantified. The endogenous ATP store, followed by PCr utilisation and oxygen-independent glycolysis contribute sequentially to the early stages of anaerobic metabolism, whilst the amount of energy supplied by each process is in the reverse order (Table 1). For elite male athletes, the maximum powers developed by oxygen-independent glycolysis, PCr utilisation and endogenous ATP utilisation were calculated as 34.1, 30.1 and 16.6 W kg$^{-1}$, respectively, with an overall maximum anaerobic power of 51.6 W kg$^{-1}$. The time constants determined for PCr utilisation, $\dot{\lambda}_2 = 20.0\ \text{s}^{-1}$ and $\psi_2 = 3.0\ \text{s}^{-1}$, are consistent with values derived by Ward-Smith (1999b). New values of $\dot{\lambda}_3 = 0.033\ \text{s}^{-1}$ and $\psi_3 = 0.34\ \text{s}^{-1}$ are proposed as offering an improved representation of the kinetics of oxygen-independent glycolysis. For the first time, tentative values for the time constants of ATP utilisation, $\dot{\lambda}_1 = 0.9\ \text{s}^{-1}$ and $\psi_1 = 20\ \text{s}^{-1}$, are suggested.

References


Hill, A.V., 1925. The physiological basis of athletic records. Presidential address to the Physiology section of the British Association for the Advancement of Science. Lancet 2, 481–486.


