

# Energy conversion strategies during 100 m sprinting

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The aims of this study were to examine energy conversion strategies during 100 m sprinting and to determine whether there are opportunities for performance enhancement beyond the usual 'maximum effort throughout' strategy. The roles of aerodynamic drag and kinetic energy recovery on the overall whole-body energy balance are discussed. A mathematical model based on sprinting with maximum effort, converting chemical energy as rapidly as possible to mechanical energy, is used to calculate a reference performance. Subsequent calculations show the effect of inserting a period of constant-speed running on overall running time. The paper explores how the timing of entry into the second phase affects the overall running time. Overall, the calculations show that no benefits result from adopting a running strategy involving the insertion of a constant-speed phase; the analysis confirms that the strategy of running with maximum effort is the optimum. For a certain range of conditions, the insertion of a short period of constant-speed running has been shown to have a negligible effect on overall running time. For elite male athletes, the constant-speed phase may be entered between 55 and 60 m from the start line, while for elite female athletes the corresponding values are between 46 and 53 m from the start.

*Keywords:* bioenergetics, biomechanics, running, sprinting.

## Introduction

The aims of this study were to examine energy conversion strategies during 100 m sprinting and to determine whether there are opportunities for performance enhancement beyond the usual 'maximum effort throughout' strategy. When examined in detail, the energy exchanges that occur during human locomotion are part of a complex process. Muscles generate mechanical energy by the conversion of chemical energy. During each stride, the mechanical energy of the muscles is used to accelerate and decelerate the arms and legs, so the kinetic and potential energy of these limbs change constantly, with some transfer of energy from one body segment to another. Muscles are capable of storing and releasing elastic strain energy. As energy exchanges take place, so part of the mechanical energy is degraded to thermal energy (heat is produced). Ultimately, having passed through several intermediate stages, the chemical energy originally released at the muscles is converted into either thermal energy (heat) or external work done by the body. The external work can be decomposed into several components: the addition of kinetic energy associated with the hori-

zontal motion of the athlete (hereafter referred to as the horizontal kinetic energy), the external work associated with the vertical movement of the body against gravity, and the external work done against aerodynamic drag. The overall energy conversion process is managed by the interaction of the athlete's feet with the ground over which he or she moves. In this paper, attention is focused on certain of the above energy contributions to determine how small changes to sprinting strategy influence overall performance.

Some years ago, Keller (1973, 1974) used optimal control theory, in conjunction with a set of equations describing the biomechanics of running, to show that a distance of 291 m is the boundary between two alternative running strategies. For distances greater than 291 m, Keller showed that the athlete should accelerate rapidly up to a constant running speed, which is then maintained over much of the rest of the race. The theory indicated that a final brief period of deceleration is required to satisfy the optimum race strategy over the full distance. Keller's calculation is largely supported by track experience. During periods of sustained running, aerobic metabolism is the principal source of energy. The optimal use of energy from the aerobic pathway is brought about by minimizing energy expenditure; this minimum corresponds to running at constant

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speed. Hence, in middle- and long-distance races, the appropriate running strategy is to accelerate quickly from rest up to a nominally constant speed that is sustained throughout the rest of the race. There is no practical experience to support the final brief period of deceleration indicated by Keller's theory. For distances less than 291 m, Keller's theory showed that the athlete should run with maximum acceleration (effort) throughout the race. However, the theory incorporated a velocity relationship that yielded a positive acceleration throughout the race for all distances up to 291 m. This is not consistent with actual experience. Competition measurements (see Moravec *et al.*, 1988) indicate that elite athletes running with maximum effort cannot sustain such an extended period of acceleration and their running speed falls away during the final phase of sprinting over distances as short as 100 m.

The introduction of sophisticated mathematical models of running based on energy considerations was pioneered by Lloyd (1967). Even before the work of Keller (1973, 1974) had been published, Lloyd and Zacks (1972) had demonstrated that only at distances below about 400 m was it profitable to sprint with maximum effort. Over longer distances, an optimized constant-speed running technique was indicated. Subsequently, various authors have used energy principles to develop the analysis of running. Ward-Smith (1985) showed that a deceleration phase during 100 m sprinting was a consequence of the limited energy available and, in more recent work, in which a three-equation model was used to describe the kinetics of anaerobic metabolism, Ward-Smith and Radford (2000a,b) reproduced the distance-time profile over the entire 100 m with considerable precision.

That it is usual for athletes to complete 100 m races by decelerating over the final phase of the race stimulated the thinking that led to the present study. The reasoning is as follows. First, it is conventional wisdom that athletes achieve their fastest time over 100 m by running with maximum effort throughout, immediately converting to mechanical energy all of the chemical energy available at the muscles. Athletes thereby attain high speeds over the ground and, as a consequence, produce excellent times over short distances. However, a simple argument shows that if the athlete can exercise appropriate control over the rate of chemical energy release, the external work done against aerodynamic drag over the final phase of the race can be modified. This contribution to the energy balance, between the point at which the maximum velocity is reached and the finishing line, is greater than the work that would be done if this phase of the race were run at a constant speed corresponding to the average over this phase. Hence, by running the final segment at an appropriate constant speed achieved by slightly modifying the time-

course of the chemical energy conversion, the possibility of improving performance is opened up. Nevertheless, there are other consequences of changing the velocity profile that must also be taken into account. First, the peak running speed is reduced, so that for part of the race the ground will be covered less rapidly. Secondly, when the final segment is run at constant speed, the horizontal kinetic energy of the runner at the finishing line will be higher than for the normal deceleration profile, which will have a metabolic cost. Hence it would only be worthwhile inserting a constant-speed phase into the running profile if the benefit gained by reducing aerodynamic drag effects outweighed the accompanying disadvantages. The energy contributions arising from the changes in the two terms under discussion, the aerodynamic drag and the kinetic energy, are small in comparison with the main terms that contribute to the overall energy balance applied to the body, but are nevertheless extremely important, as sprint races are frequently decided by small margins.

In this paper, I focus on the sprint running of elite male and female athletes to establish whether there is scope for performance enhancement over a distance as short as 100 m by varying the normal (maximum effort) energy conversion strategy. The paper tries to combine modern mathematical analysis, based on whole-body energy exchanges, with what is practicable on the running track. Two alternative sprinting strategies are investigated. In the first analysis, I assume that the athlete runs with maximum effort throughout, immediately using all chemical energy as it becomes available at the muscles. Previous mathematical models of sprinting (Lloyd, 1967; Ward-Smith, 1985, 1999a; Peronnet and Thibault, 1989; Ward-Smith and Mobey, 1995; Ward-Smith and Radford, 2000a) have implicitly assumed that, when the whole-body energy balance is calculated, the change in the horizontal kinetic energy between the peak and final running speeds is fully recoverable. Here I investigate the effect of incomplete or partial recovery on running time. In the second analysis, I assume that the athlete divides the sprint into two or three phases. The first phase is run with maximum effort, as in the first analysis. Before the maximum speed of which the athlete is capable is reached, the second phase – which is run at constant speed – is entered. In this second phase, not all of the available chemical energy is converted, but at the same time less energy is used to do external work against aerodynamic drag. If the athlete does not reach the finishing line while running at constant speed, a short deceleration phase at maximum effort completes the race. I explore how the timing of entry into the second phase affects the overall running time. The results will be compared with results obtained using the method of Ward-Smith and Radford (2000a,b), where it was

assumed that the athlete runs with maximum effort throughout the race, and the change in kinetic energy between the peak and final running speeds is fully recoverable.

## Methods

The bulk of the present analysis is based on the work of Ward-Smith and Radford (2000a), but with some important detailed changes and additions. The energy,  $C$ , released by chemical conversion at the muscles passes through a number of intermediate stages and, in conformity with the first law of thermodynamics, is ultimately transformed into external work,  $W$ , expended on the centre of mass of the runner, or degraded into heat,  $H$ . The mathematical model is simplified by considering the broad changes of energy, ignoring the cyclical variations associated with the stride pattern. Power and energy terms are expressed throughout per unit body mass.

The power equation for running can be written in the form:

$$\frac{dC}{dt} = \frac{dH}{dt} + \frac{dW}{dt} \quad (1)$$

where  $t$  is the time from the start of running. The left-hand side of equation (1) represents the rate of chemical energy conversion, and the first and second terms on the right-hand side are the rate of degradation of mechanical energy into thermal energy and the rate of external mechanical working. The net contribution to the power equation associated with the basal metabolic rate is zero and is omitted from the analysis.

Three components contribute to the rate of external working. The first two are the rate of working against aerodynamic drag,  $Dv/m$ , and the rate of working against the weight of the athlete's body,  $gdh_v/dt$ . Here  $m$  is body mass,  $v$  is the speed of the athlete over the ground,  $D$  is aerodynamic drag,  $h$  is the distance of the athlete's centre of mass above the ground and  $g$  is the gravitational acceleration. The third term requiring consideration is the contribution from the rate of change of the horizontal kinetic energy,  $dE_K/dt$ . In the present analysis, I treat this term in a manner different from previous analyses (Lloyd, 1967; Ward-Smith, 1985, 1999a; Peronnet and Thibault, 1989; Van Ingen Schenau *et al.*, 1991; Ward-Smith and Mobey, 1995; Ward-Smith and Radford, 2000a), where implicitly it was assumed that if the speed peaks and the human centre of mass is decelerated, then the change in the horizontal kinetic energy is fully recovered. Here, when  $dv/dt$  is positive, the rate of increase of the horizontal kinetic energy,  $dE_K/dt$ , is, as before, equal to  $v dv/dt$

and is taken to be positive. However, when  $dv/dt$  is negative,  $dE_K/dt$  is here taken as  $\delta(v dv/dt)$ , where  $\delta$  has a value between 0 and 1. Therefore, the rate of external mechanical working (per unit body mass) is:

$$\frac{dW}{dt} = \frac{Dv}{m} + g \frac{dh_v}{dt} + \frac{dE_K}{dt} \quad (2)$$

The drag,  $D$ , is given by:

$$D = \frac{1}{2} \rho (v - V_w)^2 S C_D = \frac{1}{2} \rho (v - V_w)^2 A_D \quad (3)$$

where  $\rho$  is the air density,  $V_w$  is the wind speed (an assisting wind being taken as positive),  $S$  is the projected frontal area of the runner,  $C_D$  is the drag coefficient and  $A_D (= S C_D)$  is the drag area.

The rate of degradation of mechanical energy is:

$$\frac{dH}{dt} = Av \quad (4)$$

where  $A$  is the rate of degradation of mechanical energy into thermal energy per unit velocity. This can be written (Ward-Smith, 1999a) as:

$$A = A_0 + A_1 V_w \quad (5)$$

where  $A_0$  and  $A_1$  are constants.

The rate of chemical energy conversion is given by:

$$\frac{dC}{dt} = \frac{dC_{an}}{dt} + \frac{dC_{aer}}{dt} \quad (6)$$

where the first and second terms on the right-hand side are the contributions from anaerobic and aerobic metabolism. Equations (1) to (6) combine to yield

$$\frac{dC_{an}}{dt} + \frac{dC_{aer}}{dt} = Av + Kv(v - V_w)^2 + g \frac{dh_v}{dt} + \frac{dE_K}{dt} \quad (7)$$

where

$$K = \frac{\rho S C_D}{2m} \quad (8)$$

The time-course of anaerobic and aerobic metabolism after the initiation of vigorous exercise is a subject of continuing debate. For example, Barstow and Mole (1991) suggested that the onset of aerobic metabolism is subject to a time delay of the order of several seconds. In the same vein, it used to be thought that the three main components of anaerobic metabolism were initiated sequentially, with different time delays. However, the view today is that these components are initiated in parallel, but with different time constants (Spriet, 1995). Restoration of the anaerobic capacity is brought about

by resynthesis, which depends on aerobic metabolism. Because of this interrelationship between the anaerobic and aerobic mechanisms, it makes sense for the body's control system to trigger the aerobic mechanism into action at the same time that the anaerobic mechanism is initiated. This argument indicates that, for aerobic metabolism as well as anaerobic metabolism, the time delay is equal to or very close to zero. Therefore, the models of aerobic and anaerobic metabolism incorporated in the present mathematical model assume zero time delay.

The early stages of aerobic energy release can be represented by the equation:

$$\frac{dC_{aer}}{dt} = R[1 - \exp(-\lambda t)] \tag{9}$$

where  $R$  is the maximum sustainable aerobic power and  $\lambda$  is a parameter governing the rate of aerobic energy release.

To describe anaerobic metabolism, I incorporate the mathematical relations recently proposed by Ward-Smith (1999b), using the suffix notation, with  $n = 1, 2, 3$ , where 1 relates to adenosine triphosphate (ATP) utilization, 2 to phosphocreatine utilization and 3 to oxygen-independent glycolysis. The power equations are of the form:

$$P_n = (P_{max})_n (r_n)^{(1/r_n)} [(1 + r_n)/r_n]^{(1+r_n)/r_n} [1 - \exp(-\psi_n t)] \exp(-\lambda_n t) \tag{10}$$

In equation (10),  $P_n$  and  $(P_{max})_n$  represent the instantaneous and maximum powers of the  $n$ th component,  $\psi_n$  and  $\lambda_n$  are time constants relating to the  $n$ th component, and

$$r_n = \frac{\psi_n}{\lambda_n} \tag{11}$$

Thus the anaerobic power is given by:

$$\frac{dC_{an}}{dt} = \sum_{n=1,2,3} (P_{max})_n (r_n)^{(1/r_n)} [(1 + r_n)/r_n]^{(1+r_n)/r_n} [1 - \exp(-\psi_n t)] \exp(-\lambda_n t) \tag{12}$$

Ward-Smith (1999b) proposed that  $\lambda_3 = \lambda$ .

The quantities  $C_{aer}$  and  $C_{an}$  are evaluated by integrating equations (9) and (12) to yield

$$C_{aer} = Rt - R[1 - \exp(-\lambda t)]/\lambda \tag{13}$$

and

$$C_{an} = \sum_{n=1,2,3} (C_{an})_n \tag{14}$$

where

$$(C_{an})_n = (P_{max})_n (r_n)^{(1/r_n)} [(1 + r_n)/r_n]^{(1+r_n)/r_n} \left[ \frac{\psi_n}{\lambda_n(\lambda_n + \psi_n)} - \exp(-\lambda_n t) \frac{\psi_n + \lambda_n [1 - \exp(-\psi_n t)]}{\lambda_n(\lambda_n + \psi_n)} \right] \tag{15}$$

During running, the nominal angle of the athlete's torso relative to the horizontal,  $\theta$ , varies during the race. Following Ward-Smith (1999a),  $\theta$  is given by the relationship:

$$\tan \theta = \frac{mg}{(mdv/dt + D)} = \frac{g}{dv/dt + K(v - V_w)^2} \tag{16}$$

Let

$$K_0 = \frac{\rho(SC_D)_0}{2m} \tag{17}$$

so that  $K$  and  $K_0$  are related by:

$$K = K_0 \sin \theta \tag{18}$$

Following Ward-Smith and Radford (2000a), the height of the centre of mass above the horizontal running surface,  $h_v$ , is computed from:

$$h_v = h_{cm} \sin \theta \tag{19}$$

where  $h_{cm}$  represents the height of the centre of mass above the ground when the athlete is standing vertically. The increase in potential energy (per unit body mass) measured relative to the condition in the starting block,  $E_h$ , is given by:

$$E_h = g(h_v - h_0) \tag{20}$$

Defining  $x$  as the distance run from the line at the start of the race, the relationship

$$\frac{dx}{dt} = v \tag{21}$$

applies universally.

At this stage, the analyses for the two strategies under consideration diverge.

### Theory 1

In this analysis, I assume that the athlete runs with maximum effort throughout the race, using both sources of energy – anaerobic and aerobic – as they become available. Defining  $x_m$  as the value of  $x$  at which the maximum speed  $v = v_{max}$  is attained, there are two phases to the analysis, depending upon whether  $x < x_m$  or  $x > x_m$ .

Phase 1:  $x < x_m$

If  $x < x_m$ , then  $dv/dt$  is positive,  $dE_K/dt$  is equal to  $v dv/dt$ , and equation (7) can be rewritten, after the substitution of equation (9), as:

$$\frac{dv}{dt} = -\frac{1}{v} [Av + Kv(v - V_w)^2 + g \frac{dh_v}{dt} - \frac{dC_{an}}{dt} - R(1 - \exp(-\lambda t))] \quad (22)$$

Phase 2:  $x > x_m$

If  $x > x_m$ , then  $dv/dt$  is negative. We write  $dE_K/dt = \delta(v dv/dt)$ , where  $\delta$  has a value between 0 and 1. The condition  $\delta = 0$  corresponds to zero kinetic energy recovery,  $\delta = 1$  corresponds to complete recovery, and values in the range  $0 < \delta < 1$  correspond to partial recovery. Integration yields  $E_K = v_{max}^2/2 - \delta(v_{max}^2/2 - v^2/2)$ . Equation (22) no longer applies and a new relationship for  $dv/dt$  is required. At this stage of the race, there are only very small changes in  $h_v$ , so I assume  $h_v$  is constant over this phase. The power equation takes one of two forms depending on whether  $\delta = 0$  or  $\delta \neq 0$ .

Phase 2a:  $x > x_m; \delta \neq 0$

For  $x > x_m$ ,  $\delta \neq 0$ , equation (22) is replaced by:

$$\frac{dv}{dt} = -\frac{1}{\delta v} [Av + Kv(v - V_w)^2 + g \frac{dh_v}{dt} - \frac{dC_{an}}{dt} - R(1 - \exp(-\lambda t))] \quad (23)$$

and the remainder of the analysis is similar to that for Phase 1.

Phase 2b:  $x > x_m; \delta = 0$

For  $x > x_m$ ,  $\delta = 0$ , the power equation is:

$$\frac{dC}{dt} = Av + Kv(v - V_w)^2 \quad (24)$$

On differentiation, this becomes:

$$\frac{d^2C}{dt^2} = A \frac{dv}{dt} + K(3v^2 - 4vV_w + V_w^2) \frac{dv}{dt} \quad (25)$$

After rearrangement, this yields:

$$\frac{dv}{dt} = \frac{d^2C/dt^2}{A + K(3v^2 - 4vV_w + V_w^2)} \quad (26)$$

Since

$$\frac{d^2C}{dt^2} = \frac{d^2C_{an}}{dt^2} + \frac{d^2C_{aer}}{dt^2} \quad (27)$$

differentiation of equations (9) and (12) yields

$$\frac{d^2C_{aer}}{dt^2} = R\lambda \exp(-\lambda t) \quad (28)$$

and

$$\frac{d^2C_{an}}{dt^2} = \sum_{n=1,2,3} (P_{ref})_n [-\lambda_n \exp(-\lambda_n t) [1 - \exp(-\psi_n t)] + \psi_n \exp(-\psi_n t) \exp(-\lambda_n t)] \quad (29)$$

where

$$(P_{ref})_n = (P_{max})_n (r_n)^{(1/r_n)} [(1 + r_n)/r_n]^{(1 + r_n)/r_n} \quad (30)$$

### Theory 1: Phases 1 and 2

When an athlete adopts the ‘set’ position in the starting blocks, the centre of mass is a distance  $h_h$  behind the starting line. The athlete’s progress along the track is found by numerical integration of equation (21), together with equation (22), (23) or (26), as appropriate. A fourth-order Runge-Kutta method was used, with a time-step of 0.01 s and between the limits  $t = 0$ ,  $x = -h_h$  and  $t = T$ ,  $x = 100$  m.

Most of the terms in the power equation are analytical and, therefore, they can be integrated exactly. Integration between the limits  $t = 0$  and  $t = T$  yields the energy equation:

$$C_{an} + C_{aer} = Ax + \int_0^T Kv(v - V_w)^2 dt + E_h + E_K \quad (31)$$

In equation (31), for  $x < x_m$ ,  $E_K = v^2/2$ ; for  $x > x_m$ ,  $E_K = v_{max}^2/2 - \delta(v_{max}^2/2 - v^2/2)$ . The aerodynamic drag term can be integrated using Simpson’s rule, while  $C_{aer}$  and  $C_{an}$  can be expressed analytically. Hence equation (31) can be used to check the growth of the residual error arising during numerical integration.

### Theory 2

During Phase 1, the athlete accelerates with maximum effort from rest,  $v = 0$ , to a speed  $v = v_c$ , where  $v_c < v_{max}$ . Phase 2 begins at  $t = t_1$ ,  $x = x_1$ ; during this second phase, the athlete maintains the constant speed  $v_c$  throughout. The constant-speed phase is concluded in one of two ways: the runner reaches the finishing line at  $t = T$ ,  $x = 100$  m; or, after time  $t = t_2$ , the available energy is less than the energy required to maintain a constant speed  $v_c$ . In the latter case, the athlete enters a third phase,

running with maximum effort and decelerating speed to the finishing line.

*Phase 1:  $v < v_c$*

The analysis for this phase, between the limits  $t = 0$  and  $t = t_1$ , is identical to that for Theory 1, Phase 1.

*Phase 2:  $v = v_c$*

This phase is entered at time  $t = t_1$  and continues until time  $t = t_2$  or  $t = T$  (see above). During this second phase, I assume that the body is capable of making available for chemical conversion a quantity of energy  $A_c$ , whereas a slightly smaller amount of energy,  $U_c$ , is actually used. I assume that  $A_c = C(t)$ , where the aerobic and anaerobic components of  $C(t)$  are given by equations (13) and (14), respectively. Thus

$$\begin{aligned} \text{For } t = t_1: & \quad U_c(t_1) = A_c(t_1) = C(t_1); & \quad x = x_1 \\ \text{For } t_1 < t < t_2: & \quad A_c(t) > U_c(t) & \quad A_c(t) = C(t) \end{aligned}$$

and

$$U_c(t) = U_c(t_1) + A(x - x_1) + K v_c (v_c - V_w)^2 (t - t_1) \quad (32)$$

$$\text{For } t = t_2: \quad U_c(t_2) = A_c(t_2) = C(t_2); \quad x = x_2$$

If entry into the second phase is delayed, the athlete may not complete the 100 m distance at constant speed, as a point will be reached where there is insufficient chemical energy available to meet demand. In this case, a third phase is necessary to complete the race.

*Phase 3:  $t_2 \leq T$*

The analysis for this phase, between the limits  $t = t_2$  and  $t = T$ , is identical to that for Theory 1, Phase 2, with  $\delta = 1$ .

*Theory 2: Phases 1, 2 and 3*

As with Theory 1, for Phase 1 the athlete's progress along the track is found by numerical integration of equations (21) and (22). A fourth-order Runge-Kutta method was used, with a time-step of 0.01 s and between the limits  $t = 0$ ,  $x = -h_h$  and  $t = t_1$ ,  $x = x_1$ . For Phase 2, a range of values for  $t_1$  was examined. The quantities  $C_{\text{aer}}$  and  $C_{\text{an}}$  were evaluated using equations (13) and (14), and  $U_c(t)$  was obtained from equation (32). The condition  $U_c(t_2) = A_c(t_2) = C(t_2)$  determines the value of  $x = x_2$  at  $t = t_2$ . If Phase 3 is required, numerical integration of equations (21) and (23) between the limits  $t = t_2$  and  $t = T$ , with  $\delta = 1$ , completes the athlete's progress.

## *Physiological and race data*

As a basis for comparing the effects of different parameter changes, a representative race performance, based on representative physiological and competition data, is required. Here the calculations are mainly based on the performance of male sprinters, but some additional calculations for female sprinters are included.

For a typical male athlete, the position of the centre of mass above the ground,  $h_v$ , is raised from its initial value ( $h_0$ ) of 0.65 m corresponding to the 'set' position in the starting blocks to about 1.0 m after some 5 m have been run (Baumann, 1976). For small values of  $t$ , some of the computed values of  $h_v$  fell in the range  $h_v < 0.65$  m; for these values,  $E_h$  was taken as zero. The horizontal distance of the centre of mass behind the starting line,  $h_h$ , varies typically from about 0.16 to 0.19 m (Baumann, 1976; Mero *et al.*, 1983). Following Ward-Smith and Radford (2000a), a constant value for  $h_h$  of 0.17 m is adopted here. To focus attention on running performance, and since it has no direct bearing on running strategy, the reaction time of the athlete was taken as zero.

In the present work, numerical values for elite male athletes from Ward-Smith and Radford (2000a) were used to compute running performance. They were derived from the average performance of the seven finalists in the 1987 World Championships. The values are as follows:

$$\begin{aligned} R &= 25 \text{ W} \cdot \text{kg}^{-1}; & A &= 3.96 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1} \\ K_0 &= 0.0029 \text{ m}^{-1}; & h_{\text{cm}} &= 1.0 \text{ m} \\ (P_{\text{max}})_3 &= 34.1 \text{ W} \cdot \text{kg}^{-1}; & \lambda_3 &= 0.033 \text{ s}^{-1}; & \psi_3 &= 0.34 \text{ s}^{-1} \\ (P_{\text{max}})_2 &= 30.1 \text{ W} \cdot \text{kg}^{-1}; & \lambda_2 &= 0.20 \text{ s}^{-1}; & \psi_2 &= 3.0 \text{ s}^{-1} \\ (P_{\text{max}})_1 &= 16.6 \text{ W} \cdot \text{kg}^{-1}; & \lambda_1 &= 0.9 \text{ s}^{-1}; & \psi_1 &= 20 \text{ s}^{-1} \end{aligned}$$

For female athletes, the following values from Ward-Smith and Radford (2000b) were adopted:

$$\begin{aligned} R &= 21.5 \text{ W} \cdot \text{kg}^{-1}; & A_0 &= 3.94 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1} \\ A_1 &= 0.0182 \text{ J} \cdot \text{s} \cdot \text{kg}^{-1} \cdot \text{m}^{-2}; & K_0 &= 0.0031 \text{ m}^{-1} \\ h_{\text{cm}} &= 0.95 \text{ m}; & h_v &= 0.62 \text{ m}; & h_h &= 0.16 \text{ m} \\ (P_{\text{max}})_1 &= 15.8 \text{ W} \cdot \text{kg}^{-1}; & \lambda_1 &= 0.9 \text{ s}^{-1}; & \psi_1 &= 20 \text{ s}^{-1} \\ (P_{\text{max}})_2 &= 21.9 \text{ W} \cdot \text{kg}^{-1}; & \lambda_2 &= 0.21 \text{ s}^{-1}; & \psi_2 &= 3.0 \text{ s}^{-1} \\ (P_{\text{max}})_3 &= 32.4 \text{ W} \cdot \text{kg}^{-1}; & \lambda_3 &= 0.041 \text{ s}^{-1}; & \psi_3 &= 0.44 \text{ s}^{-1} \end{aligned}$$

Still air conditions,  $V_w = 0$ , were assumed for both male and female performance calculations.

## **Results**

Table 1 contains two sets of results. Reference performances for elite male and female athletes running under zero wind conditions are given. Running time and instantaneous running speed are set down at 10 m

**Table 1.** Representative running performances of elite male and female 100 m sprinters under zero wind conditions

Distance (m)	Males		Females	
	Time (s)	Speed ( $\text{m} \cdot \text{s}^{-1}$ )	Time (s)	Speed ( $\text{m} \cdot \text{s}^{-1}$ )
10	1.75	8.70	1.86	8.11
20	2.80	10.11	2.98	9.46
30	3.76	10.79	4.00	10.09
40	4.67	11.16	4.98	10.38
50	5.55	11.35	5.94	10.49
60	6.43	11.41	6.89	10.46
70	7.31	11.39	7.85	10.35
80	8.19	11.30	8.83	10.17
90	9.08	11.17	9.81	9.96
100	9.98	11.01	10.83	9.73

Note: The results are given at 10 m intervals along the track. Calculations are based on Theory 1, with Phase 2 evaluated for  $\delta = 1$ . For men, the following values were adopted:  $R = 25 \text{ W} \cdot \text{kg}^{-1}$ ;  $A_0 = 3.94 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ ;  $A_1 = 0.0182 \text{ J} \cdot \text{s} \cdot \text{kg}^{-1} \cdot \text{m}^{-2}$ ;  $K_0 = 0.0029 \text{ m}^{-1}$ ;  $h_{\text{cm}} = 1 \text{ m}$ ;  $h_v = 0.65 \text{ m}$ ;  $h_h = 0.17 \text{ m}$ ;  $\lambda_3 = 0.033 \text{ s}^{-1}$ ;  $\psi_3 = 0.34 \text{ s}^{-1}$ ;  $\lambda_2 = 0.20 \text{ s}^{-1}$ ;  $\psi_2 = 3.0 \text{ s}^{-1}$ ;  $\lambda_1 = 0.9 \text{ s}^{-1}$ ;  $\psi_1 = 20 \text{ s}^{-1}$ ;  $(P_{\text{max}})_1 = 16.6 \text{ W} \cdot \text{kg}^{-1}$ ;  $(P_{\text{max}})_2 = 30.1 \text{ W} \cdot \text{kg}^{-1}$ ;  $(P_{\text{max}})_3 = 34.1 \text{ W} \cdot \text{kg}^{-1}$ ;  $V_w = 0$ . For female athletes, the following values were adopted:  $R = 21.5 \text{ W} \cdot \text{kg}^{-1}$ ;  $A_0 = 3.94 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ ;  $A_1 = 0.0182 \text{ J} \cdot \text{s} \cdot \text{kg}^{-1} \cdot \text{m}^{-2}$ ;  $K_0 = 0.0031 \text{ m}^{-1}$ ;  $h_{\text{cm}} = 0.95 \text{ m}$ ;  $h_v = 0.62 \text{ m}$ ;  $h_h = 0.16 \text{ m}$ ;  $\lambda_3 = 0.041 \text{ s}^{-1}$ ;  $\psi_3 = 0.44 \text{ s}^{-1}$ ;  $\lambda_2 = 0.21 \text{ s}^{-1}$ ;  $\psi_2 = 3.0 \text{ s}^{-1}$ ;  $\lambda_1 = 0.9 \text{ s}^{-1}$ ;  $\psi_1 = 20 \text{ s}^{-1}$ .

**Table 2.** Influence of kinetic energy recovery factor,  $\delta$ , on the overall running time of elite male 100 m sprinters under zero wind conditions

$\delta$	Overall running time (s)	$\Delta T(\text{s})$
0	10.07	0.086
0.1	10.05	0.069
0.2	10.04	0.055
0.3	10.02	0.043
0.4	10.01	0.033
0.5	10.01	0.025
0.6	10.00	0.018
0.7	9.99	0.013
0.8	9.99	0.008
0.9	9.98	0.004
1.0	9.98	0

Note: Calculations are based on Theory 1, with Phase 2 evaluated for  $0 \leq \delta \leq 1$ . The remaining parameters are as for Table 1.

intervals along the track. The calculations assume sprinting with maximum effort throughout the entire 100 m and with full kinetic energy recovery. The male performance results in Table 1 are used as a basis of comparison against which other performances considered in this paper are assessed.

In Table 2, the influence of kinetic energy recovery on overall running time is given for elite male athletes. Changes of  $\delta$  from 0 (no recovery) through a range of values of  $\delta$  corresponding to partial recovery to  $\delta = 1$  (full recovery) are considered. Kinetic energy recovery is shown to contribute up to 0.086 s of improvement to

overall running time. This contribution is less than 1%, but it is nevertheless important in the context of the close finishes typical in sprint events.

For male athletes, the effect of inserting a period of constant-speed running on overall running time is considered in Table 3. Before entry into the constant-speed phase, the athlete is assumed to sprint with maximum effort. If the athlete enters the constant-speed phase before about  $x = 40 \text{ m}$ , the remainder of the race can be completed at constant speed. However, when entry into the constant-speed phase is delayed beyond about  $x = 40 \text{ m}$ , the athlete cannot complete the entire race at constant speed. Table 3 deals with the case in which the athlete enters the constant-speed phase for a limited period and is required by energy considerations to complete the 100 m course using a short deceleration phase. The mathematical model assumes the final stage after the constant-speed phase is run with maximum effort and full kinetic energy recovery. The times at which the runner enters and emerges from the constant-speed phase and the magnitude of the constant speed are given in Table 3. For elite male athletes, the critical conditions for entry into this running strategy occur between about  $x = 40 \text{ m}$  and  $x = 60 \text{ m}$ . If the athlete enters into the constant-speed phase before a distance of about 40 m has been covered, the remainder of the race can be run at constant speed. This case is considered in Table 4, which gives results for elite male sprinters. Overall running time and the magnitude of the constant speed for a range of entry conditions are presented. The results contained in Table 4 show that

**Table 3.** Influence of a constant-speed phase on the overall running time of elite male 100 m sprinters under zero wind conditions

$x_1$ (m)	$t_1$ (s)	$t_2$ (s)	$T$ (s)	$v_c$ ( $\text{m} \cdot \text{s}^{-1}$ )	$\Delta T$ (s)
41.73	4.82	10.02	10.02	11.20	0.040
43.73	5.00	9.49	10.00	11.25	0.023
45.98	5.20	8.96	9.99	11.29	0.012
48.25	5.40	8.48	9.99	11.32	0.006
50.51	5.60	8.05	9.98	11.35	0.003
52.79	5.80	7.68	9.98	11.38	0.002
55.06	6.00	7.35	9.98	11.39	0
57.34	6.20	7.06	9.98	11.40	0
59.63	6.40	6.80	9.98	11.41	0
61.91	6.60	6.61	9.98	11.41	0

Note: Calculations are based on Theory 2, with  $\delta = 1$ . The remaining parameters are as for Table 1. The results are for a strategy involving a constant-speed phase inserted between two phases run with maximum effort. The symbols are as follows:  $x_1$  = distance from the starting line to the point at which the constant-speed phase is entered;  $t_1$  and  $t_2$  are the times at entry to and exit from the constant-speed phase;  $T$  = overall running time;  $v_c$  = speed during the constant-speed phase.

**Table 4.** Influence of a constant-speed phase on the overall running time of elite male 100 m sprinters under zero wind conditions

$x_1$ (m)	$t_1$ (s)	$T$ (s)	$v_c$ ( $\text{m} \cdot \text{s}^{-1}$ )
32.63	4.00	10.17	10.91
33.72	4.10	10.15	10.96
34.82	4.20	10.13	11.00
35.92	4.30	10.11	11.04
37.03	4.40	10.09	11.08
38.14	4.50	10.07	11.11
39.25	4.60	10.05	11.14
40.37	4.70	10.04	11.17
41.49	4.80	10.02	11.20

Note: Calculations are based on Theory 2, with  $\delta = 1$ . The remaining parameters are as for Table 1. The results are for the case where the initial phase is run with maximum effort, followed by a constant-speed phase that is maintained until the full 100 m is completed. The symbols are as follows:  $x_1$  = distance from the starting line to the point at which the constant-speed phase is entered;  $t_1$  is the time at entry to the constant-speed phase;  $T$  = overall running time;  $v_c$  = speed during the constant-speed phase.

elite male sprinters gain no advantage by adopting a strategy involving an extended constant-speed phase. Even the more refined strategy considered in Table 3 is shown to bring no advantage when compared to running with maximum effort throughout. The time differences are small (0.04 s or less) but the potential advantage of reducing aerodynamic drag is more than outweighed by the other factors at play.

Up to this stage, all of the comparisons have been based on the running profile representative of elite male athletes. Table 5 contains results based on a different running profile, that representative of elite female sprinters, as given in Table 1. Table 5 shows the effect of

inserting a period of constant-speed running on overall running time. The results may be compared with the corresponding results in Table 3 for male athletes.

## Discussion

Previous analyses (Lloyd and Zacks, 1972; Keller, 1973, 1974) had shown that, for optimum performance over 100 m, an athlete should run throughout with maximum effort. However, since those analyses were performed, advances have occurred in the mathematical modelling of sprinting. Keller's theoretical work incorporated a velocity relationship that yielded a positive acceleration throughout the race, a profile that is inconsistent with race performance. The analysis of Lloyd and Zacks was based on energy considerations and, over the intervening years, various detailed improvements, particularly the treatment of anaerobic energy conversion, have led to much more accurate modelling of the velocity time-course over 100 m. The present analysis was undertaken to determine whether a more sophisticated mathematical model of sprinting would lead to conclusions different from those reached by Keller (1973, 1974) and Lloyd and Zacks (1972). In particular, sprinting performance has been studied to establish whether the insertion of a constant-speed phase into the speed profile leads to improved performance.

The representative data in Table 1 are fully consistent with the mathematical model of Ward-Smith and Radford (2000a,b). For male and female athletes, the data were derived from the average performance of the finalists of the men's and women's 100 m sprints in the 1987 World Championships. The present calculations exclude the effects of reaction time. This is

**Table 5.** Influence of a constant-speed phase on the overall running time of elite female 100 m sprinters under zero wind conditions

$x_1$ (m)	$t_1$ (s)	$t_2$ (s)	$T$ (s)	$v_c$ ( $\text{m} \cdot \text{s}^{-1}$ )	$\Delta T$ (s)
30.97	4.10	10.91	10.92	10.13	0.084
31.98	4.20	10.51	10.89	10.17	0.060
34.02	4.40	9.78	10.86	10.24	0.031
36.08	4.60	9.14	10.85	10.30	0.016
38.14	4.80	8.57	10.84	10.35	0.008
40.22	5.00	8.07	10.84	10.39	0.004
42.30	5.20	7.63	10.84	10.42	0.002
44.38	5.40	7.25	10.83	10.45	0.001
46.48	5.60	6.91	10.83	10.47	0
48.57	5.80	6.62	10.83	10.48	0
50.67	6.00	6.36	10.83	10.49	0
52.77	6.20	6.21	10.83	10.49	0

Note: Calculations are based on Theory 2, with  $\delta = 1$ . The remaining parameters are as for Table 1. The results are for a strategy involving a constant-speed phase inserted between two phases run with maximum effort. The symbols are as follows:  $x_1$  = distance from the starting line to the point at which the constant-speed phase is entered;  $t_1$  and  $t_2$  are the times at entry to and exit from the constant-speed phase;  $T$  = overall running time;  $v_c$  = speed during the constant-speed phase.

the short delay that occurs during races, between the instant at which the starting gun and the measuring equipment are simultaneously activated and the time at which a sprinter reacts to the starting signal. For comparison with the elite men's and women's results computed by Ward-Smith and Radford (2000a,b), and the corresponding race measurements for the 1987 World Championships, reaction times of 0.18 and 0.19 s, respectively, must be added to the present calculations to derive the overall running time.

Current mathematical models of sprinting based on energy considerations (Lloyd, 1967; Ward-Smith, 1985, 1999a; Peronnet and Thibault, 1989; Ward-Smith and Mobey, 1995; Ward-Smith and Radford, 2000a,b) assume implicitly that the metabolic cost of increasing the horizontal kinetic energy of the runner depends only on the athlete's speed at the finishing line, and is independent of the speed profile over the remainder of the track. Expressed another way, the calculations implicitly assume that all of the body's horizontal kinetic energy lost from the point of maximum speed to that at the end of the race is fully recoverable in a form that reduces the metabolic cost of running over this segment of the race. The recent analysis of Ward-Smith and Radford (2000a), incorporating the assumption of full recovery, provides strong support for this view, giving computed results for the time-course at 10 m intervals along the track that are in excellent agreement with measured data, with a root-mean-square error of less than 0.01 s. Unpublished calculations show that the use of values of  $\delta$  less than 1 lead to poorer agreement between computed and track data, with larger root-mean-square errors. The effect on overall running time of kinetic energy recovery is shown in Table 2.

There is no obvious external mechanism by which an athlete can control the value of  $\delta$  during running. What the calculations demonstrate is that the human body has highly effective internal mechanisms for minimizing energy dissipation associated with energy transformations involving this term in the energy balance.

For male sprinters, the calculations show that, if entry into a constant-speed phase is delayed until some 60 m have been run, the athlete reaches his peak speed; thereafter, it is not possible to convert sufficient chemical energy to sustain a period of sprinting at constant speed. The performance corresponds to sprinting with maximum effort throughout. Comparing the results of Tables 3 and 4, it is evident that there is no advantage to be gained by entering the constant-speed phase too early. If the second phase is entered too soon, although there will be sufficient chemical energy available, the athlete will take too long to complete the specified distance because the constant speed is lower than the optimal running speed.

Tables 2–4 are based on a running profile representative of an elite male athlete. As is evident from Table 1, elite female athletes have a rather different running profile. Female sprinters reach their peak running speed some 10 m sooner than male sprinters and the subsequent decline in running speed is rather more pronounced. Calculations based on a running profile representative of elite female sprinters, showing the effect of inserting a period of constant-speed running on overall running time, are contained in Table 5; these may be compared with the results for men in Table 3. The results in Table 5 confirm that the strategy of running with maximum effort is the best available.

Although the conclusions of this paper are based on two sets of representative biophysical parameters (i.e. for elite male and for elite female sprinters), I am confident that the results are of general applicability to elite athletes. Individual athletes will have running profiles that differ, to a greater or lesser extent, from the representative profiles of the present models. Nevertheless, for a given class of athlete (e.g. elite male sprinter), the overall effect of the broad similarities between individual profiles far outweighs the effects of the relatively small differences. This has been proved, for example, by the extensive amount of experience built up on the generalized application of wind-correction factors (see Ward-Smith, 1999a), which are used to provide an adjustment to overall running time to allow for wind effects on sprinting performance.

Overall, the calculations show no benefits result from adopting a running strategy involving the insertion of a constant-speed phase. The analysis confirms that running with maximum effort remains the optimum strategy. For male sprinters, if the constant-speed phase is entered within about 5 m of the point of normal peak running speed – values of  $x_1$  between about 55 and 60 m – then the insertion of a period of constant-speed running has a negligible effect on overall running time (Table 3). For female sprinters, because peak running speed is reached nearer the starting line, the corresponding values of  $x_1$  are between about 46 and 53 m. It is possible that athletes may be able to extract some benefit from these periods if they can use the time to relax briefly to focus on exerting maximum effort in the closing stages of the sprint.

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